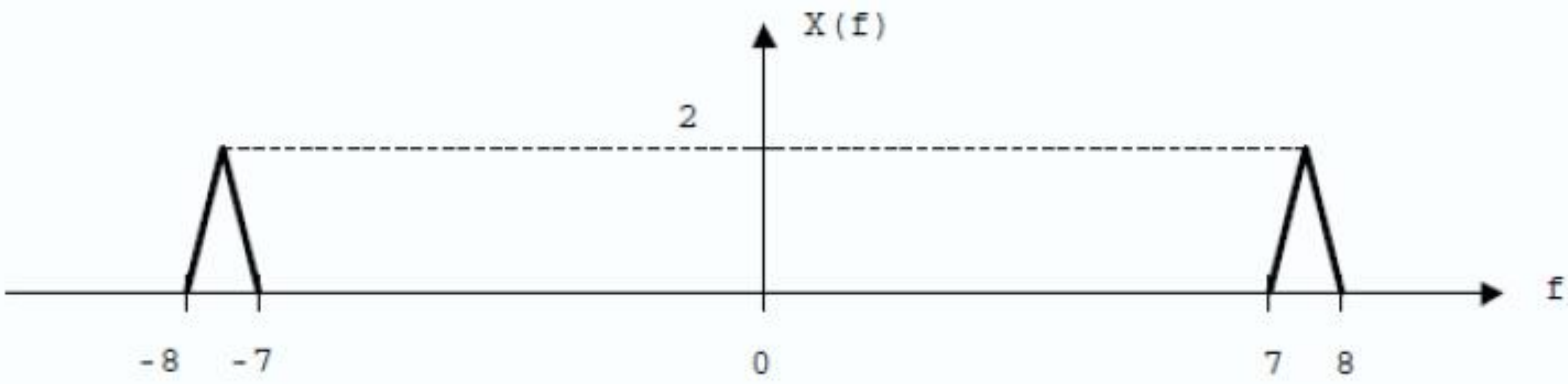
Some homeworks

Exercise #1

Consider the bandpass signal $x(t)$ whose spectrum is shown below.



This signal is sampled at $fs = 4$ samples/second with ideal impulses.

(a) Sketch the spectrum $X_\delta(f)$ of the sampled signal

(b) The sampled signal is now passed through an ideal *lowpass* filter (LPF) of bandwidth $W = 1$ and amplitude $1/fs$. What is the relation between the output of the LPF and the original signal? In other words, the cascade of downsampling and low pass filtering, to which operation is equivalent?

(c) Sketch the spectrum of the complex envelope of the output of the LPF and write its expression in time c(t).

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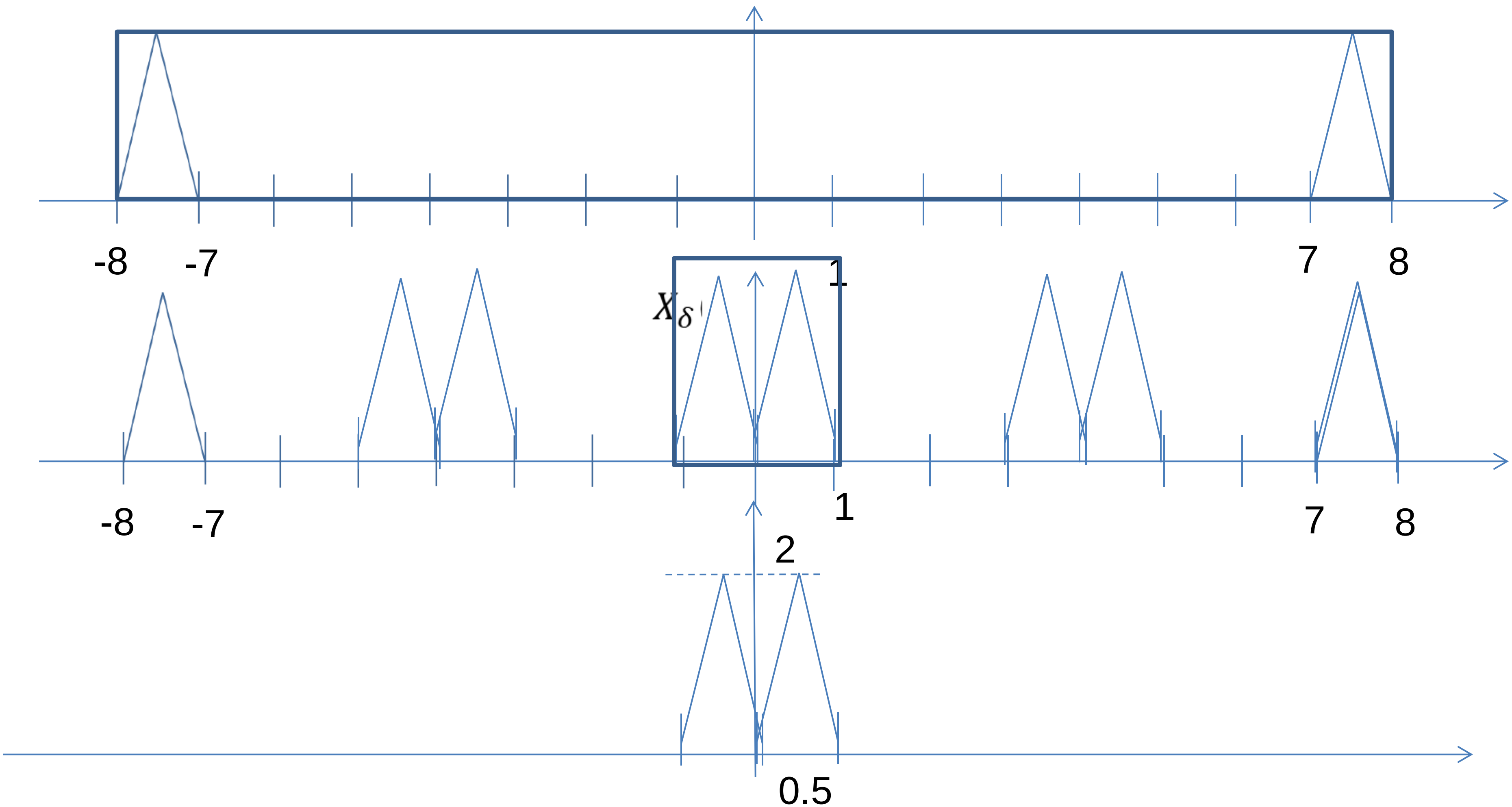
Solution

$X_\delta(f)$ is the periodic repetition of the original spectrum, repeated each 4Hz



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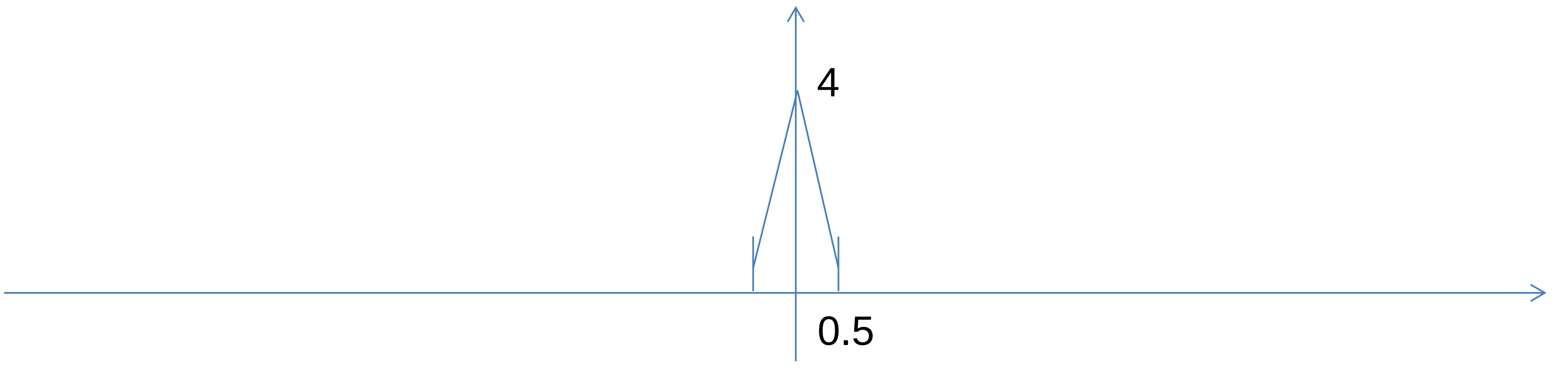
Solution

By downsampling (i.e., sampling at a frequency lower than the Nyquist frequency), and filtering, we have performed a frequency downconversion. The resulting signal is a passband signal with central frequency 0.5

What is the complex envelope of the resulting signal after low pass filtering?



$$C(f) = 4rect\left(\frac{f}{0,5}\right) * rect\left(\frac{f}{0,5}\right)$$

$$\Downarrow$$

$$c(t) = 4\left(0,5sinc\left(\frac{t}{2}\right) \cdot 0,5sinc\left(\frac{t}{2}\right)\right) = sinc^2(t/2)$$

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Solution

$$s(t) = a(t)cos\omega_c t - b(t)sin\omega_c t$$

$$sin(a) - \sin(b) = 2cos\frac{a+b}{2}sin\frac{a-b}{2}$$

$$100(sin(\omega_c + \omega_a)t - \sin(\omega_c - \omega_a)t = 2cos\omega_c t sin\omega_a t$$

$$s(t) = 200cos\omega_c t sin\omega_a t + 500\ cos\omega_c t = a(t)cos\omega_c t$$

$$\downarrow$$

$$\mathrm{a}(t) = 200sin\omega_a t + 500$$

$$\mathrm{b}(t) = 0$$

$$c(t) = a(t) + jb(t) = a(t)$$

$$m(t) = 200/500sin\omega_a \mathrm{t}$$

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Homework:
Find the complex envelope of a passband filter

$$G(f) = rect\left(\frac{f - f_c}{2B}\right) + rect\left(\frac{f + f_c}{2B}\right)$$

Solution:
The analytic signal is given by $z(t) = 2g_+$ where g_+
is the part of g(t) that has a spectral extension in f>0.

$$\Downarrow$$

$$G_+(f) = rect\left(\frac{f - f_c}{2B}\right)$$

$$\Downarrow$$

$$Z(f) = 2G_+(f) = 2rect\left(\frac{f - f_c}{2B}\right)$$

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The spectrum of the complex envelope is translated to the left of fc

$$C(f) = 2G_{+}(f + f_c) = 2rect\left(\frac{f}{2B}\right)$$

$$c(t) = 4B\sin c(2Bf)$$