Homework: Let us consider the stationary process n(t) with power spectral density:

$$\boxed{W_n(f) = W_0 rect\left(\frac{f - f_c}{2B}\right) + W_0 rect\left(\frac{f + f_c}{2B}\right)}$$

Calculate spectral density of z, c, a, b and the related statistical powers.

$$\boxed{W_z(f) = 4W_+(f) = 4W_0 rect\left(\frac{f - f_c}{2B}\right)}$$

$$\boxed{W_c(f) = 4W_+(f + f_c) = 4W_0 rect\left(\frac{f}{2B}\right)}$$

$$\boxed{a = \frac{1}{2}(c + c^*) \qquad b = \frac{1}{2}(c - c^*)}$$

$$R_a(\tau) = \frac{1}{4} E[(c(t) + c^*(t))(c^*(t+\tau) + c(t+\tau))]$$

$$= \frac{1}{4} E[c(t)c(t+\tau)] + E[c^*(t)(c^*(t+\tau)]$$

$$= \frac{1}{4} R_c(\tau) + \frac{1}{4} R_{c^*}(\tau)$$

Nota: c e il suo coniugato sono ortogonali

$$R_b(\tau) = \frac{1}{4} E[(c(t) - c^*(t))(c^*(t+\tau) - c(t+\tau))] = \frac{1}{4} R_c(\tau) + \frac{1}{4} R_{c^*}(\tau)$$

$$W_a(f) = W_b(f) = \frac{1}{4} W_c(f) + \frac{1}{4} W_c(-f) = 2W_0 rect\left(\frac{f}{2B}\right)$$

$$M_c = M_z = 2M_v = 2(4W_0 B) = 8W_0 B$$

$$M_a = M_b = M_v = 4W_0 B$$

homework

Given the complex envelope $c(t) = V_0 e^{j\phi_0}$

Find the spectral density of the passband signal of which c(t) is the complex envelope in both hypotheses

Case a): ϕ_0 *is* a uniform variable between $(0, 2\pi)$

Case b): ϕ_0 a constant

Solution:
Case a)
In case a), c(t) is a stationary random process
Moreover c is orthogonal to his conjugate

$$R_{cc^*}(\tau) = E[c(t)c(t+\tau)] = E[V_0 e^{j\phi_0} V_0 e^{j\phi_0}] = V_0^2 E[e^{j2\phi_0}] =$$
$$V_0^2 \frac{1}{2\pi}\int_0^{2\pi} e^{j2v} dv = 0$$

$\Downarrow$

$c(t), c^*(t)$ are orthogonal and in this case:

$$\boxed{W_v(f) = \frac{1}{2}[W_c(f - f_0) + W_c(-f - f_0)]}$$

where

$$\boxed{W_c(f) = F[R_c(\tau)]}$$

$$\boxed{R_c(\tau) = E[c(t)c^*(t+\tau)] = V_0^2}$$

$$\boxed{W_c(f) = V_0^2\delta(f)}$$

$$\boxed{W_v(f) = \frac{1}{2}V_0^2[\delta(f - f_0) + \delta(-f - f_0)]}$$

In case b)

$$R_{cc^*}(\tau) = E[c(t)c(t+\tau)] = E[V_0 e^{j\phi_0} V_0 e^{j\phi_0}] = V_0^2 e^{j2\phi_0} \neq 0$$

c(t) is not orthogonal to his conjugate and the passband signal v(t) is cyclostationary

homework

Consider a generic linear amplitude modulated signal

$$v(t) = a(t)\cos(\omega_0 t + \phi_0)$$

Where a(t) is stationary and the phase of the carrier uniformly distributed random variable. Moreover, the bandwidth of a(t) is lower than the carrier frequency.
Find the spectral density of v(t)

Solution:

In theory, we should calculate the autocorrelation:

$$R_v(t,\tau) = E[v(t+\tau)v^*(t)]$$

and then, once verified that is is stationary, then we can calculate the spectral density as its Fourier Transform.
However, it is more convenient to use the concept of complex envelope.
Thanks to the theorem about the product of two random process/signals, we can state that:
$z_v(t) = a(t)z_b(t)$

where $b(t) = \cos(\omega_0 t + \varphi_0)$

homework

$$c_v(t) = a(t)c_b(t) = a(t)e^{j\varphi_0}$$

$$\Downarrow$$

$$R_c(\tau) = R_a(\tau)$$

$$\Downarrow$$

$$W_c(f) = W_a(f)$$

$$\Downarrow$$

$$\boxed{W_v(f) = \frac{1}{4}\{W_a(f - f_0) + W_a(-f - f_0)\}}$$

Homework:

Find the SQNR for a signal uniformly distributed on [-1,1]
and quantized by a uniform quantizer with 256 levels.

Solution:

$$SQNR = \frac{E\left[X^2\right]}{E\left[(x - Q(x))^2\right]}$$

The sample of the signal is a random variable uniformly distributed on [-1,1]

$$\Downarrow$$

$$E\left[X^2\right] = \frac{1}{2}\int_{-1}^{1} dv = 1$$

Assuming the quantization error uniformly distributed on [-D/2,D/2] where D is the quatization step

$$E\left[(x - Q(x))^2\right] = \frac{D^2}{12}$$

$$E\left[(x-Q(x))^2\right]=\frac{D^2}{12}=\frac{a^2}{3\cdot 4^{\upsilon}}$$

Where ν=8
a=1

$$SQNR=10\log_{10}\left(\frac{1}{1/3\cdot 4^{\upsilon}}\right)=10\log_{10}3+10\upsilon\log_{10}4$$

$$=4.77+80\cdot\log_{10}4=4.77+48.16=52.93dB$$