- **Exercise #1**
- The random process V*(t)* is defined by

$$V(t) = X cos 2\pi f_0 t + Y sin 2\pi f_0 t$$

- where *X* and *Y* are two zero-mean independent Gaussian random variables each with the variance σ^2
- Find the expected value of *V(t).*
- Find the autocorrelation . Is V*(t)* stationary?
- Find the power spectral density of *V(t)*
- Find the power spectral density of the complex envelope of *V(t)*
- Find the in-phase and quadrature components of *V(t)*

•**Exercise #1**

•Expected value of *V(t):* $E[V(t)] = E[X]cos2\pi f_0 t + E[Y]sin2\pi f_0 t = 0$

•Find the autocorrelation

$R_v(\tau) = E[V(t)V^*(t+\tau)] =$

$\mathrm{E}[(Xcos2\pi f_0 t + Ysin2\pi f_0 t)(Xcos2\pi f_0(t+\tau) + Ysin2\pi f_0(t+\tau))] =$

$E[X^2cos2\pi f_0 tcos2\pi f_0(t+\tau) + Y^2sin2\pi f_0 tsin2\pi f_0(t+\tau)] +$

$E[XYcos2\pi f_0 tsin2\pi f_0(t+\tau) + XYsin2\pi f_0 tcos2\pi f_0(t+\tau)]$=

•$E[X^2]cos2\pi f_0 tcos2\pi f_0(t+\tau) + E[Y^2]sin2\pi f_0 tsin2\pi f_0(t+\tau) +$

$E[XY]cos2\pi f_0 tsin2\pi f_0(t+\tau) + E[XY]sin2\pi f_0 tcos2\pi f_0(t+\tau)$

Zero, as they are independent

•$cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$

•$R_v(\tau) = \sigma^2 cos2\pi f_0 \tau$ $\Rightarrow$ V(t) is stationary

- **Exercise #1**
- Find the power spectral density of *V(t)*
- Find the power spectral density of the complex envelope of *V(t)*
- Find the in-phase and quadrature components of *V(t)*

- $R_v(\tau) = \sigma^2 cos2\pi f_0 \tau$

$$\Downarrow$$

- $W_v(f) = \frac{\sigma^2}{2}(\delta(f - f_0) + \delta(f + f_0))$
- $W_c(f) = 4W_{v^+}(f + f_0) = 2\sigma^2\delta(f)$
- $v(t) = a(t)cos\omega_0 t - b(t)sin\omega_0 t$ $\Rightarrow$ General relationship between in-phase and in quadrature components and the passband signal

$$V(t) = Xcos2\pi f_0 t + Ysin2\pi f_0 t \Rightarrow a(t) = X \quad b(t) = -Y$$

Exercise #2

A stationary random process has an autocorrelation function given by

$$R_x(\tau) = \frac{A^2}{2} e^{|\tau|} cos2\pi f_0 \tau$$

We know that the random process never exceeds 6 in magnitude, e.g. assume that A=6.

How many quantization levels are required to guarantee an SQNR of at least 60dB?

Assuming that the signal is quantized to satisfy the condition above, and assuming the approximate bandwidth of the signal is W, what is the minimum required bandwidth for the transmission of a binary PCM signal based on this quantization scheme?

Exercise #2

How many quantization levels are required to guarantee an SQNR of at least 60dB?

$$SQNR = 10log_{10}\frac{P_X}{a^2} + 6v + 4.8$$

$$P_X = R_x(0) = \frac{A^2}{2}$$

$$a = A$$

$$SQNR = 10log_{10}\frac{A^2}{A^2 2} + 6v + 4{,}8 \geq 60$$

$$v \geq \frac{60 - 4{,}8 + 3}{6} = 9{,}7$$

$$v = 10$$

$$N = 2^{10} = 1024$$

Exercise #2

How many quantization levels are required to guarantee an SQNR of at least 60dB?

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$$v = 10 \Rightarrow N = 2^{10} = 1024$$

Exercise #2

Assuming that the signal is quantized to satisfy the condition above, and assuming the approximate bandwidth of the signal is W, what is the minimum required bandwidth for the transmission of a binary PCM signal based on this quantization scheme?

Sampling frequency $f_s = 2W$

$$\Downarrow$$

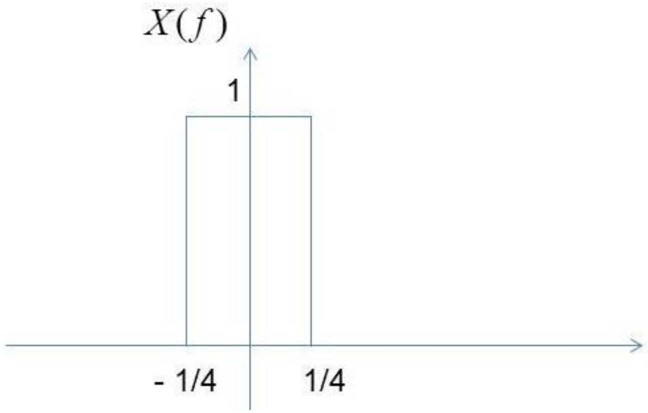
$$R_s = R_b = f_s v = 10 \cdot 2W$$

$$\Downarrow$$

$$B = 10 \cdot \frac{2W}{2} = 10W$$

Exercise #3

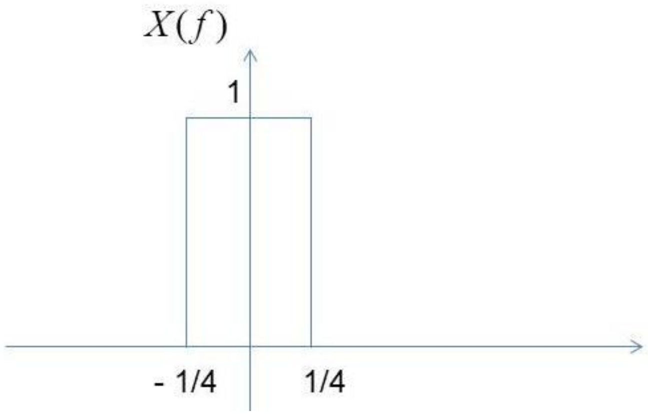
The spectrum of a lowpass signal is shown below:



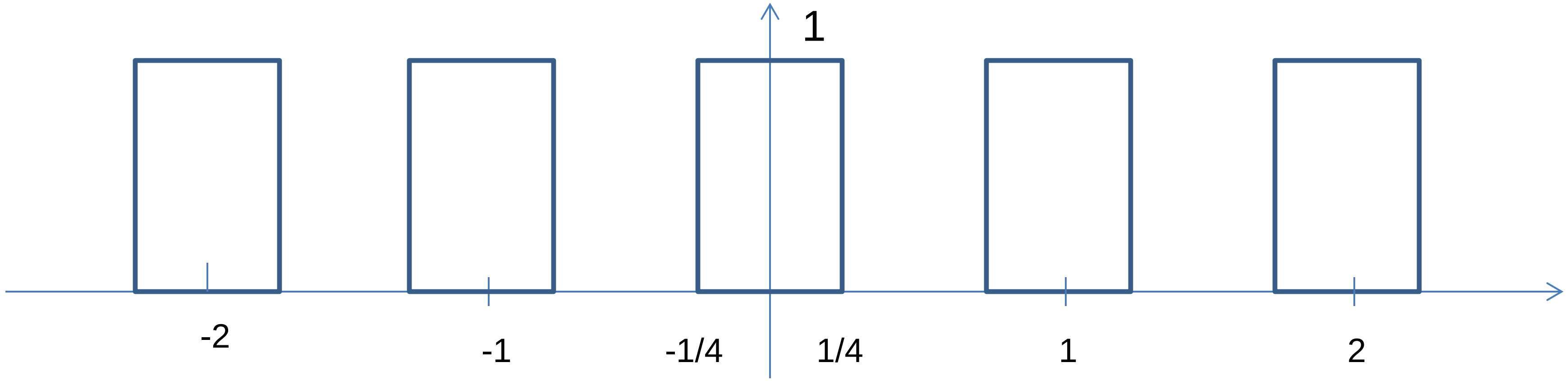
Determine the value of the Nyquist sampling rate, fs,min.
Sketch the ideal sampled spectrum Xδ(f), for the sampling rate fs = 2fs,min
Sketch the spectrum of the complex envelope of the output of an ideal (rectangular) bandpass filter with f0 = 2, B = 0.6, and gain 1/4.

Exercise #3

The spectrum of a lowpass signal is shown below:

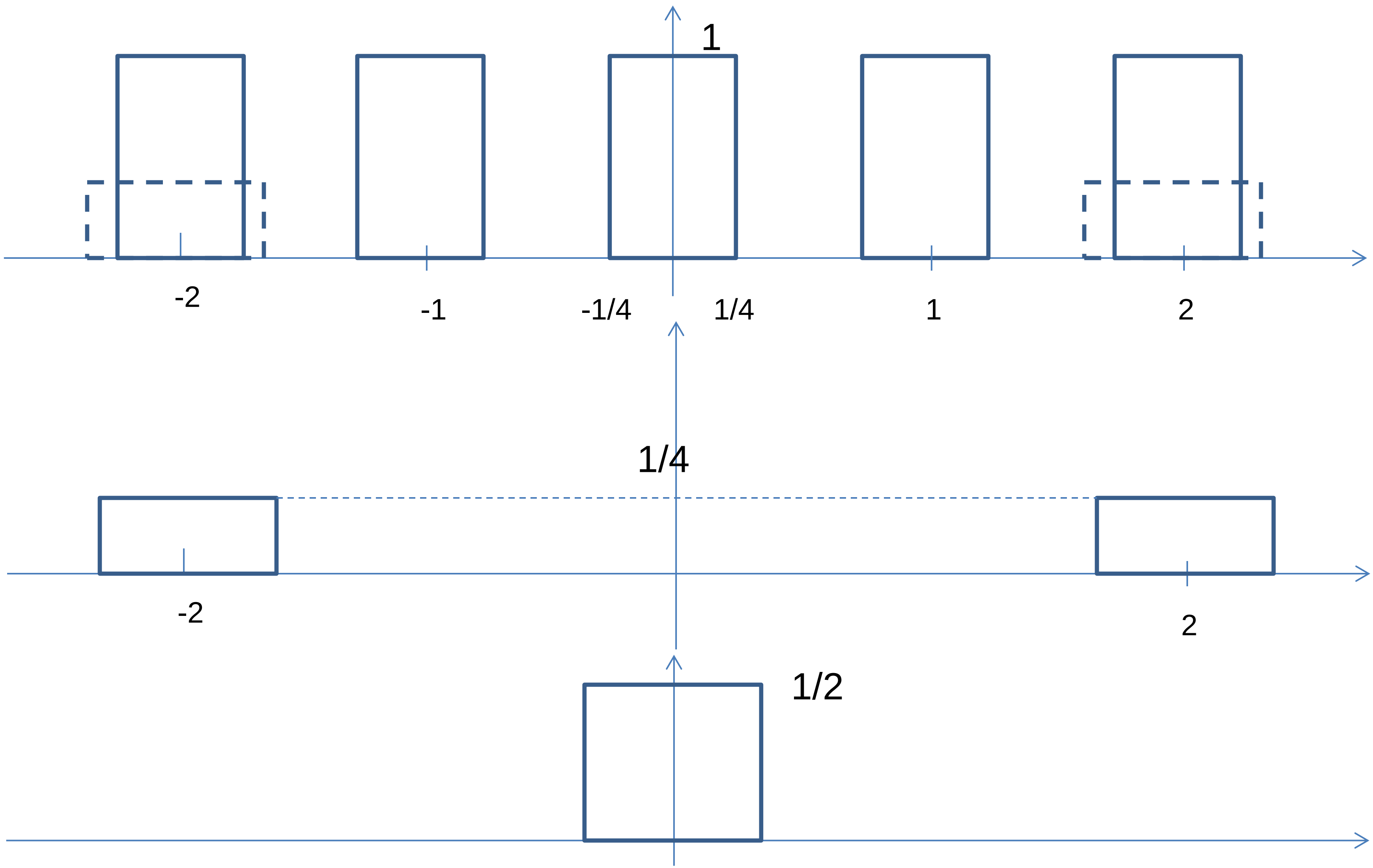


Determine the value of the Nyquist sampling rate, fs,min=2/4=0,5
Sketch the ideal sampled spectrum Xδ(f), for the sampling rate fs = 2fs,min=1



Exercise #3

Sketch the spectrum of the complex envelope of the output of an ideal (rectangular) bandpass filter with f0 = 2, B = 0.6, and gain 1/4.



Exercise #4

A message signal m(t) is transmitted by binary PCM. Let the signal to-quantization noise (SQNR) required be at least 46 dB. Determine the minimum number of bit required to encode each sample, assuming that m(t) is sinusoidal. With this value of quantization levels, determine the SQNR.

$$SQNR = 10log_{10}\frac{P_X}{a^2} + 6v + 4.8$$

Let us assume that the amploitude of the sinusoidal waveform is A

$$P_X = \frac{A^2}{2}$$

$$a = A$$

$$SQNR = 10log_{10}\frac{A^2}{A^2 2} + 6v + 4,8 \geq 46$$

$$v \geq \frac{46 - 4,8 + 3}{6} = 7,3$$

$v = 8$

$$SQNR = -3 + 6 \cdot 8 + 4,8 = 55,8dB$$