Digital Communications
Bandpass Modulation

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BANDPASS MODULATION

In case of digital bandpass modulation, digital symbols are transformed into waveforms that take the form of shaped pulses that MODULATES a sinusoid called CARRIER wave or simply CARRIER.

In radio transmission the carrier is converted to an EM field for propagation

In digital communications, the modulation process corresponds to switching or keying the **amplitude, frequency**, or **phase** of a sinusoidal carrier wave according to incoming digital data

Three basic digital modulation techniques:

- ❑Amplitude-shift keying (ASK) -special case of AM
- ❑Frequency-shift keying (FSK) -special case of FM
- ❑Phase-shift keying (PSK) -special case of PM

We can also use combination of them, such ASK-PSK modulation (an example is the Quadrature Amplitude Modulation, QAM)

<u>Will use signal space approach in receiver design and performance analysis</u>

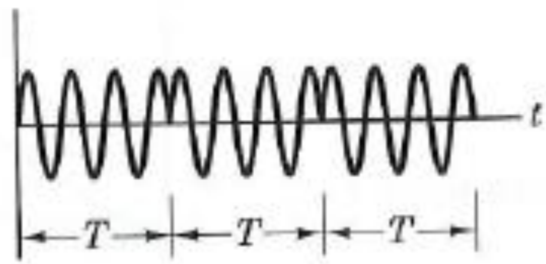
BANDPASS MODULATION
PHASE SHIFT KEYING

Analytic

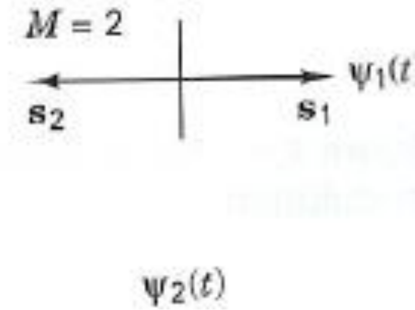
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + 2\pi i/M)$$

$i = 1, 2, \ldots, M$
$0 \le t \le T$

Waveform



Vector

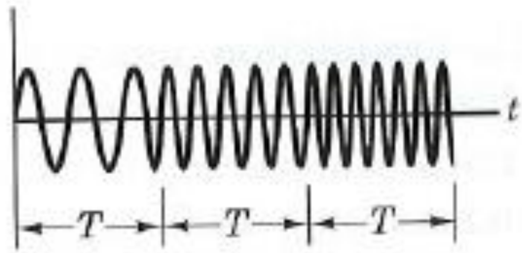


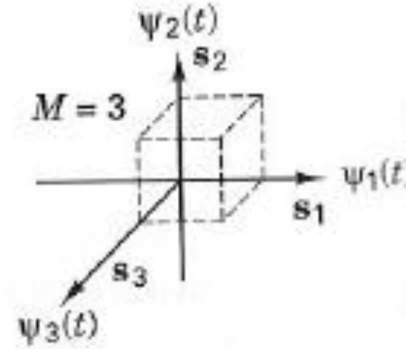
BANDPASS MODULATION

FREQUENCY SHIFT KEYING

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos(\omega_i t + \phi)$$

$$i = 1, 2, \ldots, M$$
$$0 \le t \le T$$





BANDPASS MODULATION

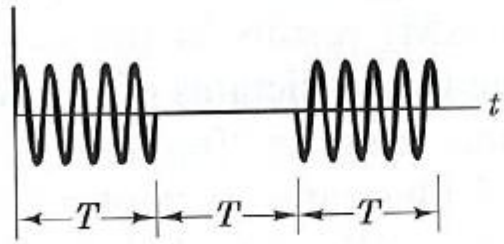
AMPLITUDE SHIFT KEYING

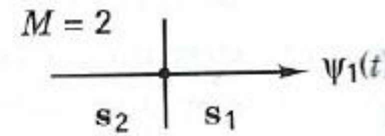
ASK

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos(\omega_0 t + \phi)$$

$i = 1, 2, \ldots, M$

$0 \le t \le T$





BANDPASS MODULATION

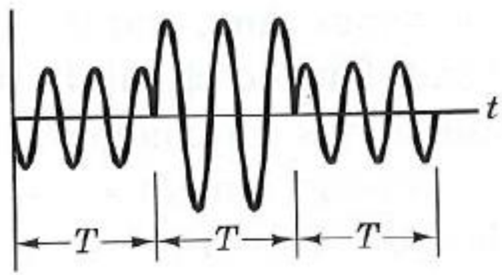
ASK/PSK

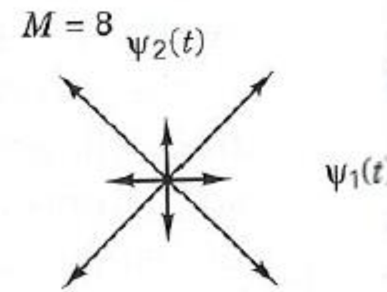
ASK/PSK (APK)

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos\left[\omega_0 t + \phi_i(t)\right]$$

$$i = 1, 2, \ldots, M$$

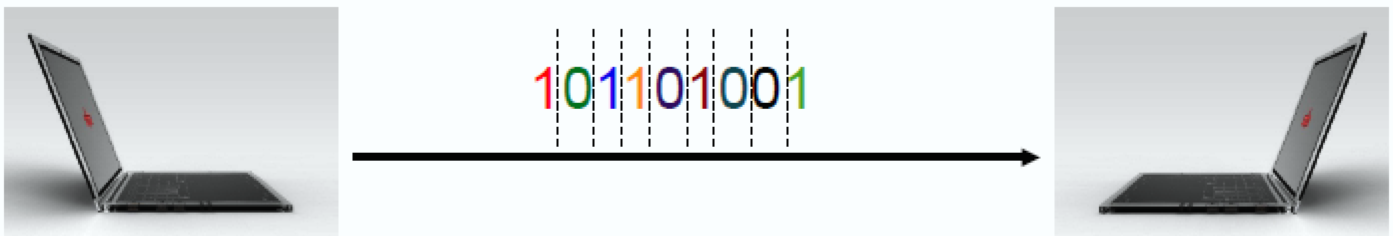
$$0 \le t \le T$$





BANDPASS MODULATION

- In binary signaling, the modulator produces one of two distinct signals in response to one bit of source data at a time.

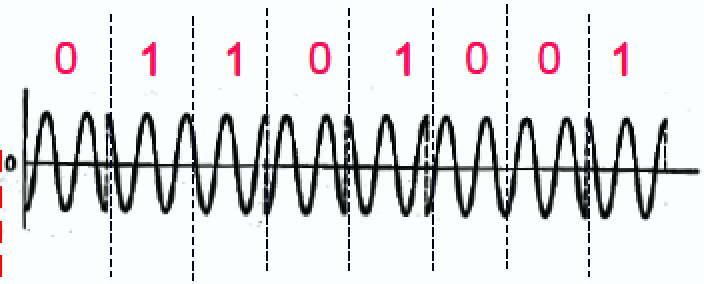


- Binary modulation types

Binary PSK | **Binary FSK** | **Binary ASK**

BANDPASS MODULATION
Binary PSK (BPSK)

- Modulation



"1" $\longrightarrow s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$

"0" $\longrightarrow s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos\boxed{(2\pi f_c t + \pi)} = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$

- $0 \leq t < T_b$, T_b bit duration
- f_c : carrier frequency, chosen to be n_c/T_b for some fixed integer n_c or $f_c >> 1/T_b$
- E_b : transmitted signal energy per bit, i.e.

$$\int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_2^2(t)dt = E_b$$

- The pair of signals differ only in a 180-degree phase shift

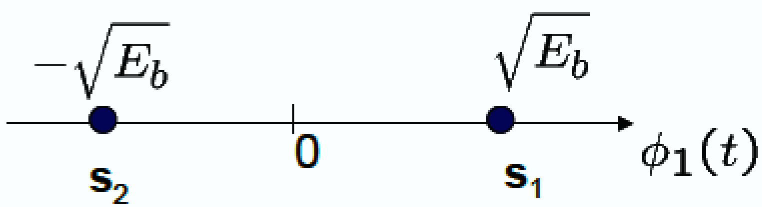
BANDPASS MODULATION
Binary PSK (BPSK)

- There is one basis function

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \quad \text{with} \quad 0 \le t < T_b$$

- Then $s_1(t) = \sqrt{E_b}\phi_1(t)$ and $s_2(t) = -\sqrt{E_b}\phi_1(t)$

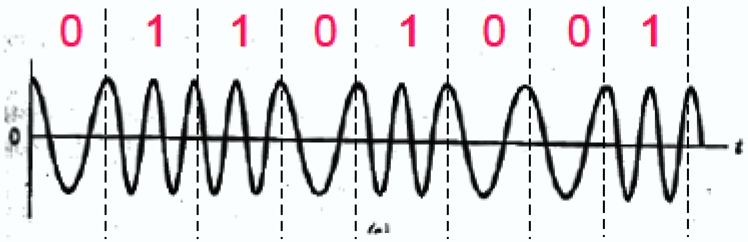
- A binary PSK system is characterized by a signal space that is one-dimensional (i.e. $N=1$), and has two message points (i.e. $M=2$)



$$d_{12} = 2\sqrt{E_b}$$

BANDPASS MODULATION
Binary FSK (2FSK)

- Modulation



"1" → $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$

"0" → $s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t)$

$0 \le t < T_b$

- E_b : transmitted signal energy per bit

$$\int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_2^2(t)dt = E_b$$

- f_i : transmitted frequency with separation $\Delta f = f_1 - f_0$
- Δf is selected so that $s_1(t)$ and $s_2(t)$ are orthogonal i.e.

$$\int_0^{T_b} s_1(t)s_2(t)dt = 0$$

(Example?)

BANDPASS MODULATION
Binary FSK (2FSK)

- Two orthogonal basis functions are required

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) \qquad 0 \le t < T_b$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \qquad 0 \le t < T_b$$

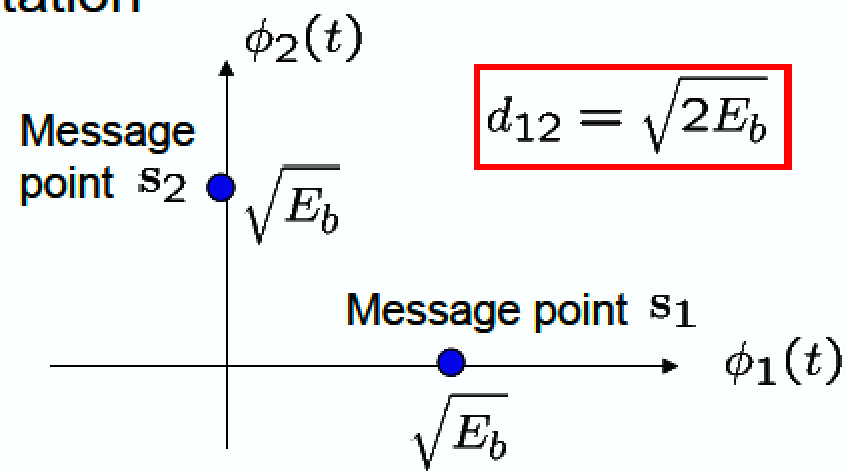
$$\Rightarrow \quad s_1(t) = \sqrt{E_b}\,\phi_1(t)$$

$$s_2(t) = \sqrt{E_b}\,\phi_2(t)$$

- Signal space representation

$$\mathbf{s}_1 = [\sqrt{E_b} \quad 0]$$

$$\mathbf{s}_2 = [0 \quad \sqrt{E_b}]$$



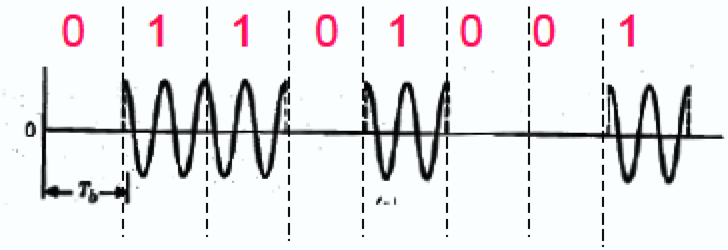
$$\boxed{d_{12} = \sqrt{2E_b}}$$

BANDPASS MODULATION
Binary ASK (2ASK)

- Modulation

"1" → $s_1(t) = \sqrt{\frac{2E}{T_b}}\cos(2\pi f_c t)$

"0" → $s_2(t) = 0 \qquad 0 \le t < T_b$



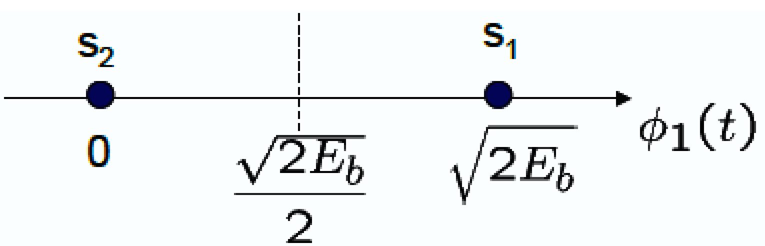
(On-off signaling)

- Average energy per bit

$$E_b = \frac{E+0}{2} \quad \text{i.e.} \quad E = 2E_b$$

Signal representation

$$d_{12} = \sqrt{2E_b}$$



BANDPASS MODULATION
Comparison for binary bandpass modulations

more eucledean distance' than fsk and ask

low probability of error

BPSK	BFSK	BASK
$d_{1,2} = 2\sqrt{E_b}$	$d_{1,2} = \sqrt{2E_b}$	$d_{1,2} = \sqrt{2E_b}$
$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$	$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

to get same perror II need to increase SNR

- In general,

$$\boxed{P_e = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)}$$

BANDPASS MODULATION
M-PSK

For a generic shape of the transmitting baseband pulse, the M bandpass waveforms are:

$$s_i(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi i}{M}\right) \quad i = 0,\ldots\ldots M-1 \quad 0 \le t \le T$$

In case the transmitting filter is a rectangular pulse:

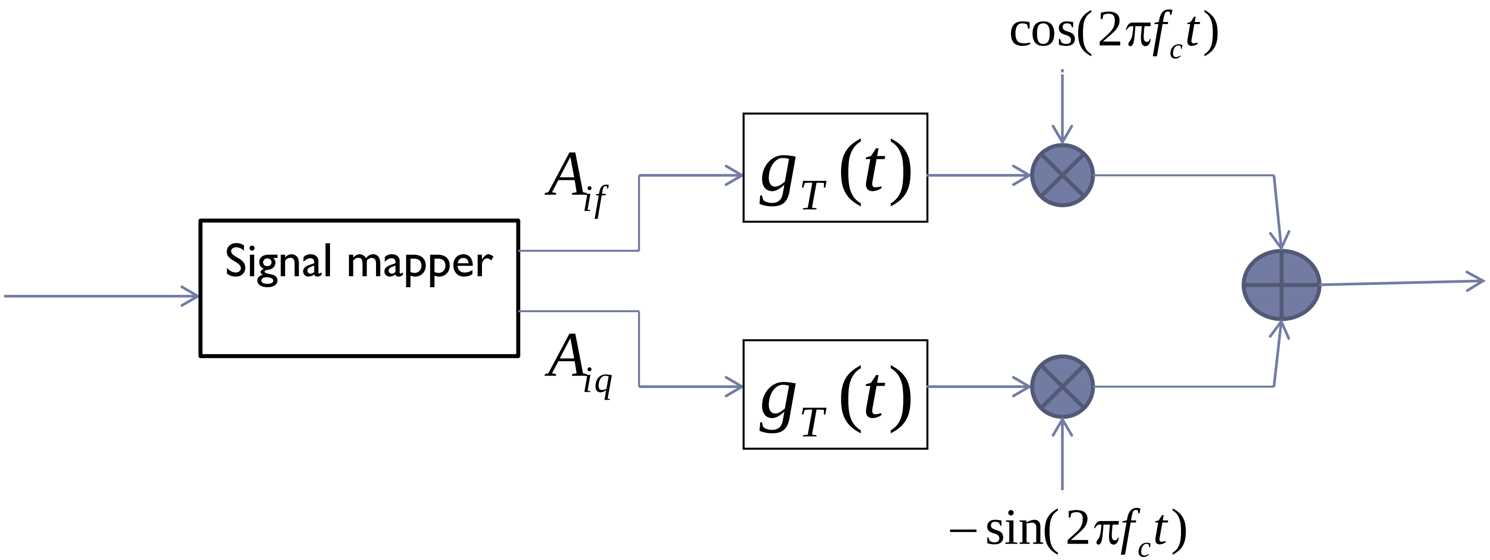
$$s_i(t) = \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{2\pi i}{M}\right) \quad i = 1,\ldots M-1 \quad 0 \le t \le T$$

$$s_i(t) = g_T(t)\cos(2\pi i/M)\cos(2\pi f_c t) - g_T(t)\sin(2\pi i/M)\sin(2\pi f_c t) =$$

$$= g_T(t)A_{if}\cos(2\pi f_c t) - g_T(t)A_{iq}\sin(2\pi f_c t)$$

$$A_{if} = \cos(2\pi i/M) \qquad A_{iq} = \sin(2\pi i/M)$$

M-PSK



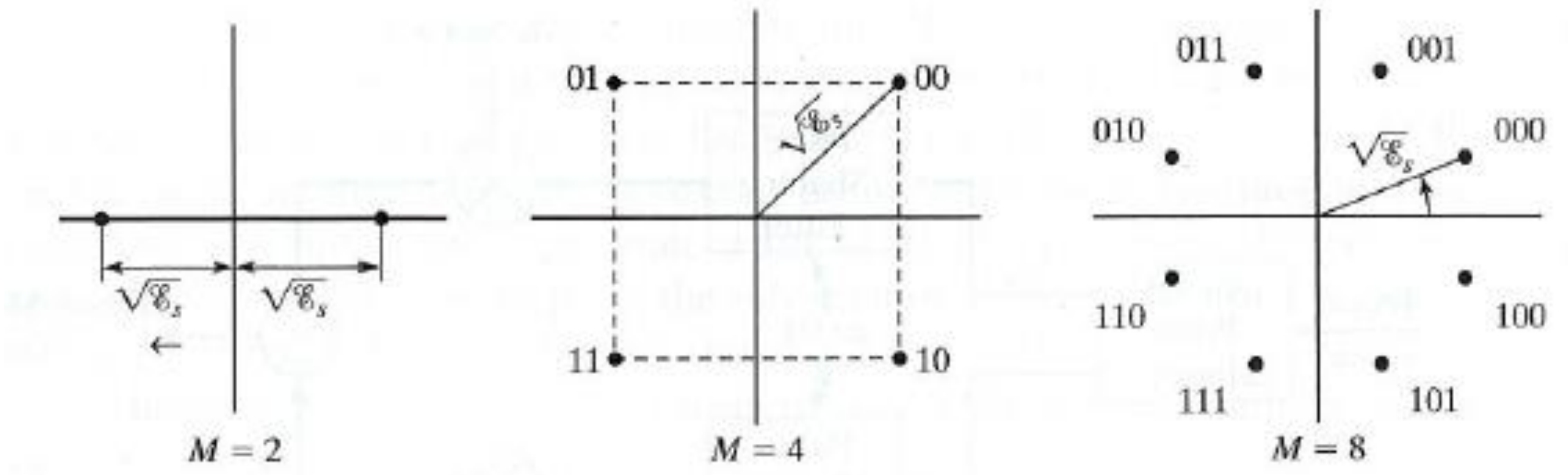
Block diagram of a digital phase-modulator

$$\psi_1(t) = \sqrt{\frac{1}{E}} g_T(t) \cos\left(2\pi f_c t\right)$$

$$\psi_1(t) = -\sqrt{\frac{1}{E}} g_T(t) \sin\left(2\pi f_c t\right)$$

$$s_i = \left(\sqrt{E}\cos\left(2\pi i / M\right), \sqrt{E}\sin\left(2\pi i / M\right)\right)$$

BANDPASS MODULATION
M-PSK



PSK signal constellations with Gray Coding (adjacent phases differ by one binary digit)
As the most likely errors caused by noise involve erroneous selection of an adjacent phase to the transmitted phase, only a single bit erro occus in the k-bit sequence with Gray coding.

BANDPASS MODULATION

M-PSK

$$d_{mn} = \sqrt{\left\| s_m - s_n \right\|^2} = \sqrt{2E\left(1 - \cos\frac{2\pi(m-n)}{M}\right)}$$

$$\Downarrow$$

$$d_{mim} = \sqrt{2E\left(1 - \cos\frac{2\pi}{M}\right)} = 2\sqrt{E}\sin\frac{\pi}{M}$$

Exercise

determine the minimum distance between adjancet points of the constellation of PSK modulation with M=2,4,8 assuming that the energy is the same E.
For M=8, determine how many dB the transmitted signal energy E must be increased to achieve the same minimum distance as M=4.

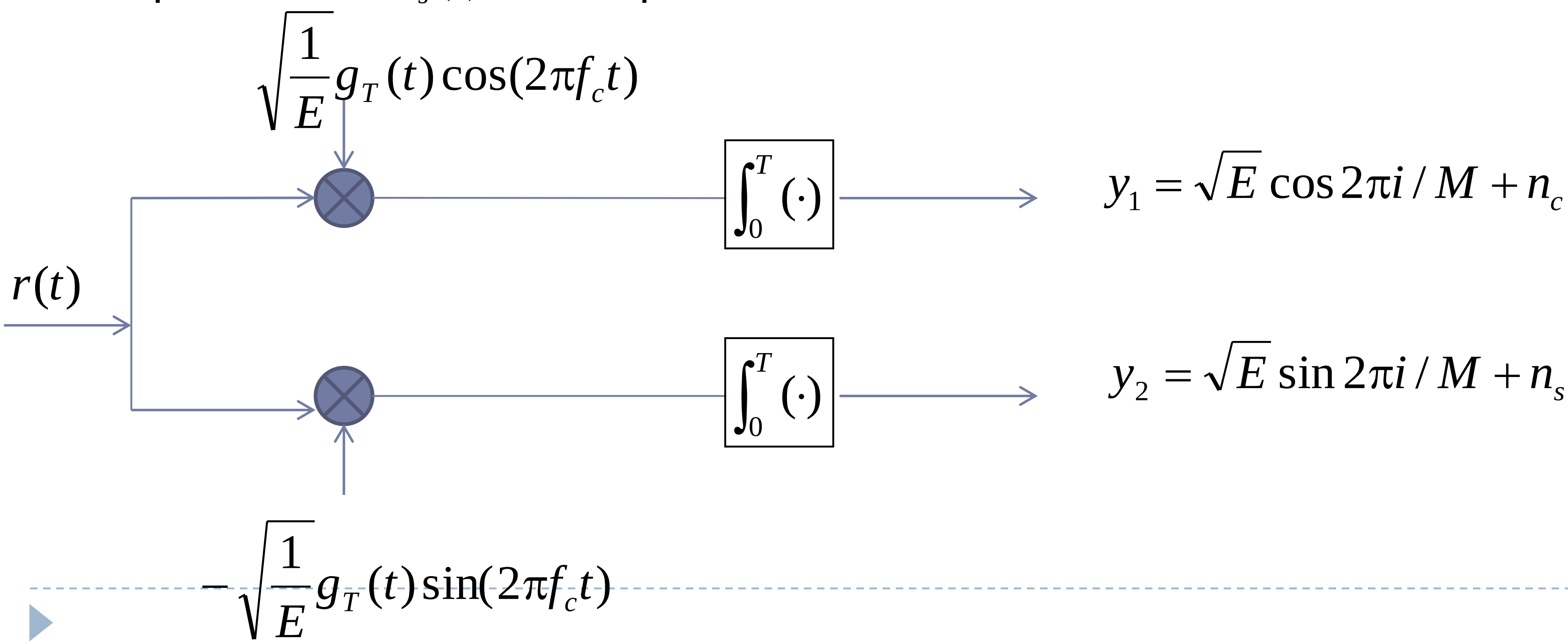
BANDPASS MODULATION

M-PSK – demodulation and detection

Assuming perfect CARRIER PHASE ESTIMATION, the received signal can be expressed as:

$$r(t) = s_i(t) + n(t) =$$
$$= (g_T(t)A_{if} + n_c(t))\cos\left(2\pi f_c t\right) - (g_T(t)A_{iq} + n_s(t))\sin\left(2\pi f_c t\right)$$

where the additive WGN has been expressed in terms of its in-phase $n_c(t)$ and quadrature $n_s(t)$ components.



M-PSK – demodulation and detection

$$n_c = \frac{1}{\sqrt{4E}} \int_0^T g_T(t) n_c(t) dt$$

$$n_s = \frac{1}{\sqrt{4E}} \int_0^T g_T(t) n_s(t) dt$$

As the in-phase and quadrature components are zero mean and uncorrelated

$$\Downarrow$$

$$E[n_c] = E[n_s] = 0 \quad \text{and} \quad E[n_c n_s] = 0$$

$$E[n_c^{\ 2}] = E[n_s^{\ 2}] = \frac{1}{4E} \int_0^T \int_0^T g_T(t) g_T(\tau) E[n_c(t) n_c(\tau)] dt d\tau = \frac{N_0}{4E} \int_0^T g_T^2(t) dt = \frac{N_0}{2}$$

M-PSK – demodulation and detection

The optimum detector projects the received signal vector onto each of the M possible transmitte signal vectors and selects the vector corresponding to the largest projection. Therefore, itr computes the correlation metrics

$$\mathbf{y} \cdot \mathbf{s_i}$$

However, as all the signals have equal energy, an equivalent detector metric for digital PSK is to compute the phase of the received signal vector $\mathbf{y} = (y_1, y_2)$

$$\boxed{\Theta = \tan^{-1} \frac{y_2}{y_1}}$$

Another possible detector metric for PSK

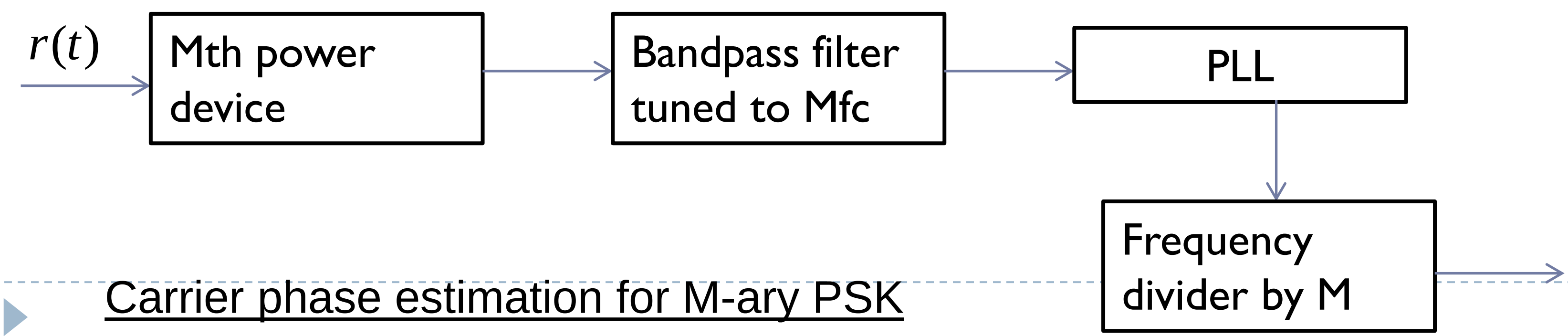
M-PSK – Carrier phase estimation

In case of **COHERENT** demodulation, we assumed that the function $\psi(t)$ is perfectly synchronized with r(t) in both TIME and CARRIER PHASE.

In practice, this does not hold as:

1) The propagation delay through the channel results in a carrier offset in the received signal
2) The oscillator that generates the carrier at the received is not phase-locked to the oscillator used at the transmitter
3) Practical oscillators drift in frequency and phase

We need to generate a phase-coherent carrier at the receiver by using the received signal.



Probability of error for coherent QPSK (4-PSK)

Basically we have two binary-phase modulated signals in phase quadrature

With perfect estimate of the carrier phase, there is no crosstalk or interference between the signals on the two quadrature carriers

The **bit error probability** is the same as the one of BPSK!

What about the symbol error probability?

Correct probability for the two-bit symbol

$$P_c = (1-P_2)^2 = \left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]^2$$

$$P_4 = 1 - P_c = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$$

Probability of error for coherent QPSK (4-PSK)

For not too low signal-to-noise ratio

$$\frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right) << 1$$

$$\Downarrow$$

$$\boxed{P_4 \approx 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

Probability of error for coherent M-PSK

Let us find an approximation for the symbol error probability for a generic M

The error probability in selecting a particular signal point other than the transmitted signal point when the signal is corrupted by AWGN is given by

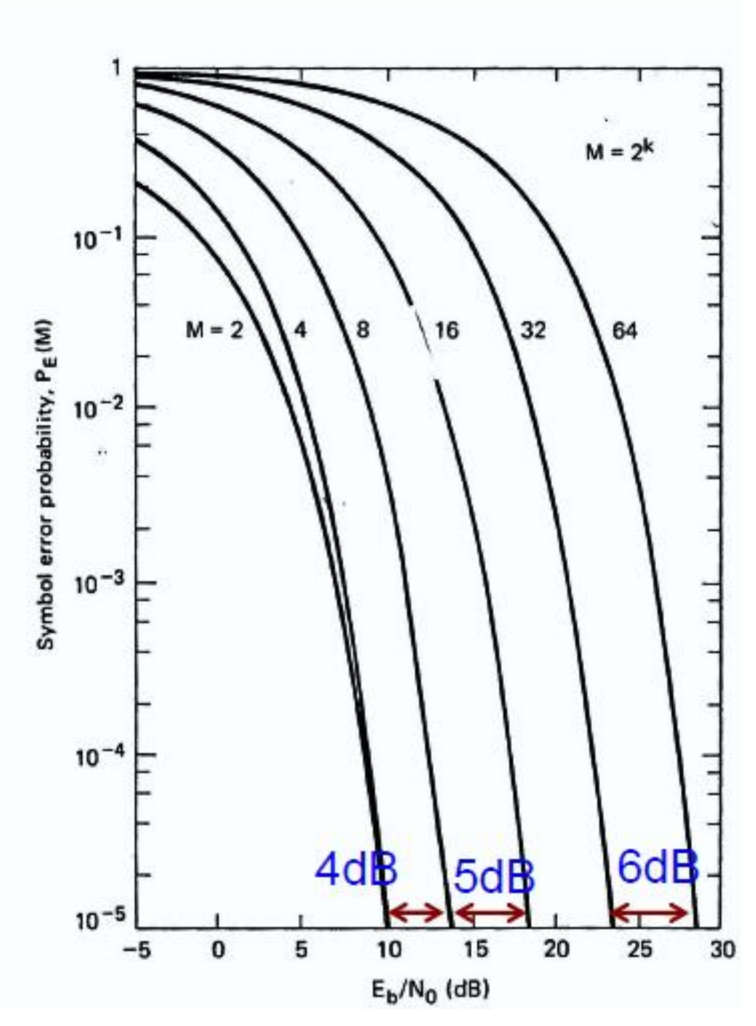
$$\boxed{P_2 = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)}$$

Where d_{12} is the square of the Euclidean distance between the transmitted signal point and the particular erroneously selected signal point.
In case of PSK, the error probability is dominated by the erroneous selection of either one of the two signal points adjacent to the transmitted signal point.

$$\boxed{P_M \approx 2Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right)} \Rightarrow \boxed{P_M \approx 2Q\left(\sqrt{2\rho_S}\sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{\frac{2\pi^2 \log_2 M}{M^2}\frac{E_b}{N_0}}\right)}$$

Doubling M deteriorates the performance by a factor of 4 (6B)

Probability of error for coherent M-PSK



Probability of error for coherent M-PSK

What about the bit error probability of a generic M-PSK?

In general, it is not straghforward to find the bit error probability from te symbol error probability as it depends from the mapping of the k-bit symbls into the corresponding signal phases.

In case of Gray coding, as most probable errors due to noise result in the erroneous selection of an adjacent phase, most k-bit symbol errors cointain only one single bit error

$$\boxed{P_b \approx \frac{1}{k} P_M}$$

Non-coherent PSK: Differential PSK

- Phase synchronization is eliminated using differential encoding
  - Encode the information in phase difference between successive signal transmission. In effect,
  - to send "0", advance the phase of the current signal by 180^0;
  - to send "1", leave the phase unchanged
- Provided that the unknown phase θ contained in the received wave varies slowly (constant over two bit intervals), the phase difference between waveforms received in two successive bit intervals will be independent of θ.

Non-coherent PSK: Differential PSK

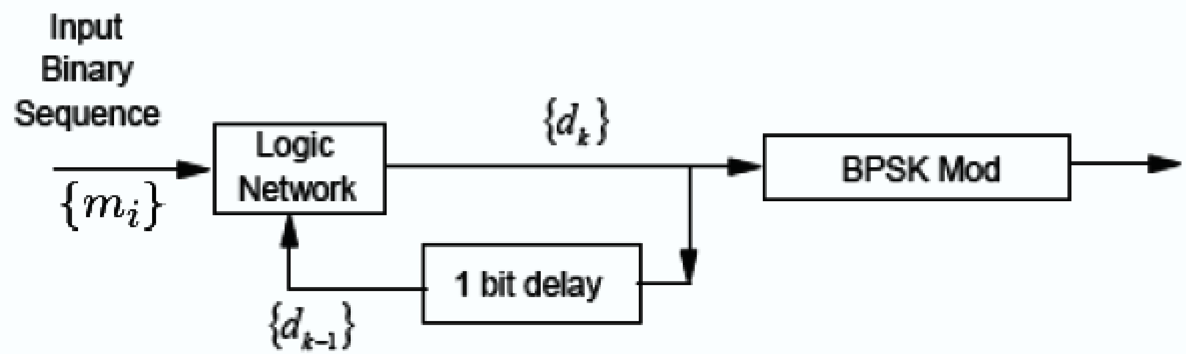
- Generate DPSK signals in two steps
  - Differential encoding of the information binary bits
  - Phase shift keying

- Differential encoding starts with an arbitrary reference bit

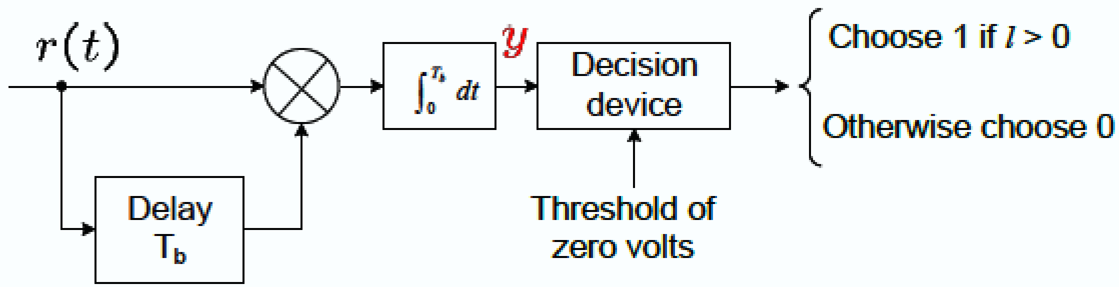
Information sequence		1	0	0	1	0	0	1	1	$\{m_i\}$
Differentially encoded sequence	1	1	0	1	1	0	1	1	1	$\{d_i\}$
	Initial bit									
Transmitted Phase	0	0	π	0	0	π	0	0	0	

$$\boxed{d_i = \overline{d_{i-1} \oplus m_i}}$$

Non-coherent PSK: Differential PSK



Non-coherent PSK: Differential PSK



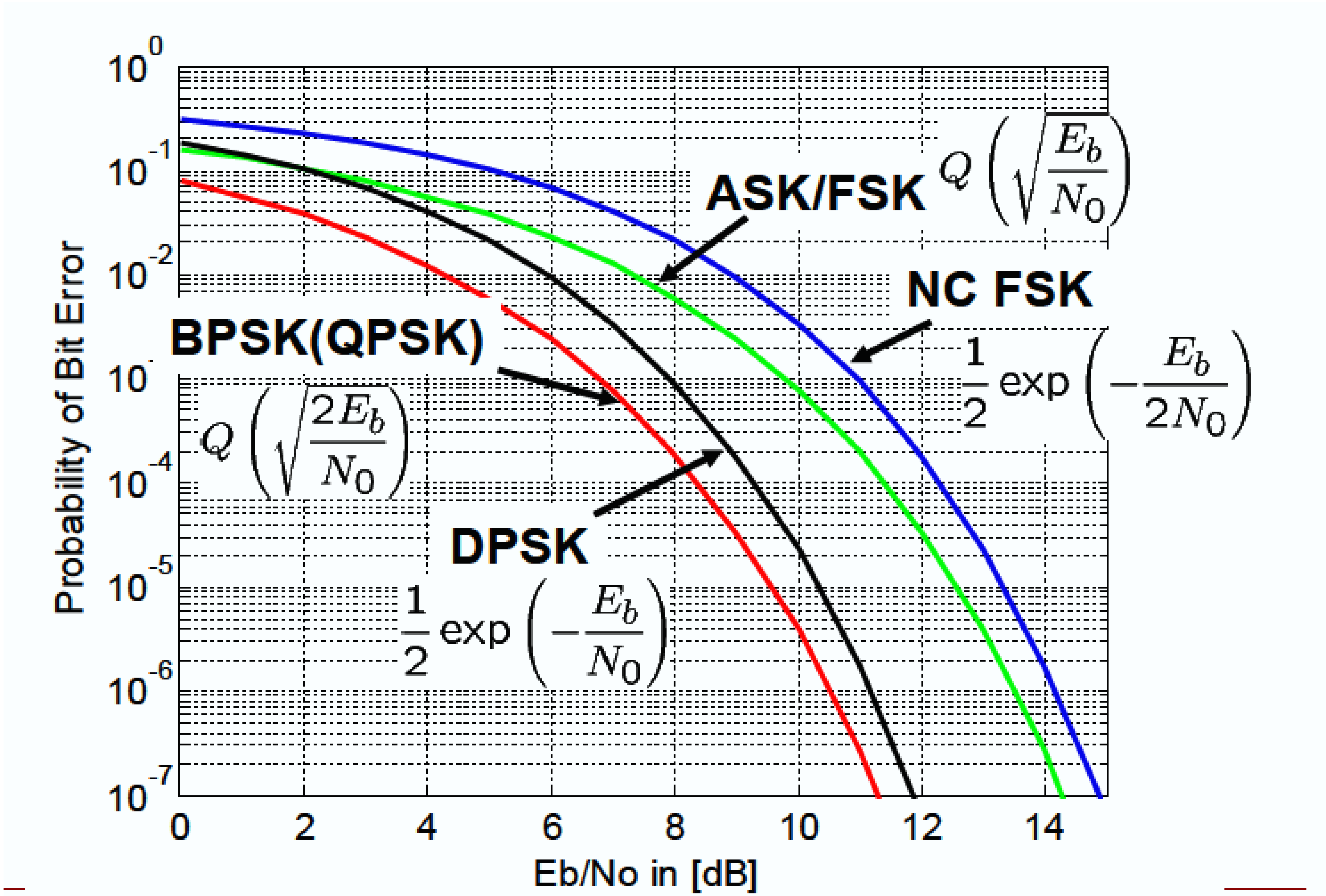
- Output of integrator (assume noise free)

$$y = \int_0^{T_b} r(t) r(t - T_b) dt = \int_0^{T_b} \cos(w_c t + \psi_k + \theta) \cos(w_c t + \psi_{k-1} + \theta) dt$$

$$\propto \cos(\psi_k - \psi_{k-1})$$

  - The unknown phase θ becomes irrelevant
  - If $\psi_k - \psi_{k-1} = 0$ (bit 1), then y > 0
  - if $\psi_k - \psi_{k-1} = \pi$ (bit 0), then y < 0
- Error performance $P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$

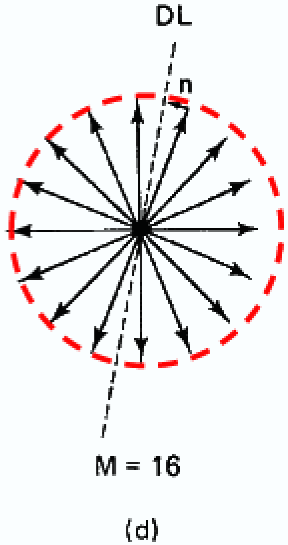
Non-coherent PSK: Differential PSK



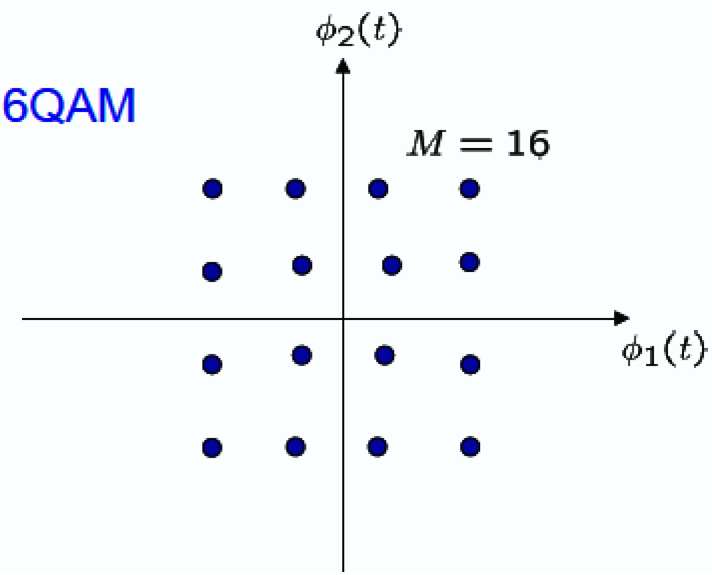
BANDPASS MODULATION
Quadrature Amplitude Modulation

- In MPSK, in-phase and quadrature components are interrelated in such a way that the envelope is constant (circular constellation)
- If we relax this constraint, we get M-ary QAM

16PSK



16QAM



BANDPASS MODULATION
Quadrature Amplitude Modulation

- Signal set:

$$s_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}} b_i \sin(2\pi f_c t) \qquad 0 \leq t < T$$

  - E_0 is the energy of the signal with the lowest amplitude
  - a_i, b_i are a pair of independent integers
- Basis functions:

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad 0 \leq t < T$$

- Signal space representation

$$\vec{s}_i = [\sqrt{E_0} a_i \quad \sqrt{E_0} b_i]$$

BANDPASS MODULATION
Quadrature Amplitude Modulation

- Upper bound of the symbol error probability

$$P_e \le 4Q\left(\sqrt{\frac{3kE_b}{(M-1)N_0}}\right) \qquad \text{(for } M = 2^k \text{)}$$

- **Exercise:**

  Determine the increase in Eb required to maintain the same error performance if the number of bits per symbol is increased from k to k+1, where k is large.

BANDPASS MODULATION
Quadrature Amplitude Modulation

It is a specific combination of ASK and PSK.
Both amplitude and/or phase changes in different symbols.
QAM is implemented by modulating two PAM baseband signals both in phase and in quadrature.
Let us consider two PAM baseband signals:

$$\sum_k a_k g_T(t-kT) \quad \text{e} \quad \sum_k b_k g_T(t-kT)$$

The transmitted bandpass signal is:

$$s(t) = \sum_k a_k g_T(t-kT)\cos\omega_0 t + \sum_k b_k g_T(t-kT)\sin\omega_0 t$$

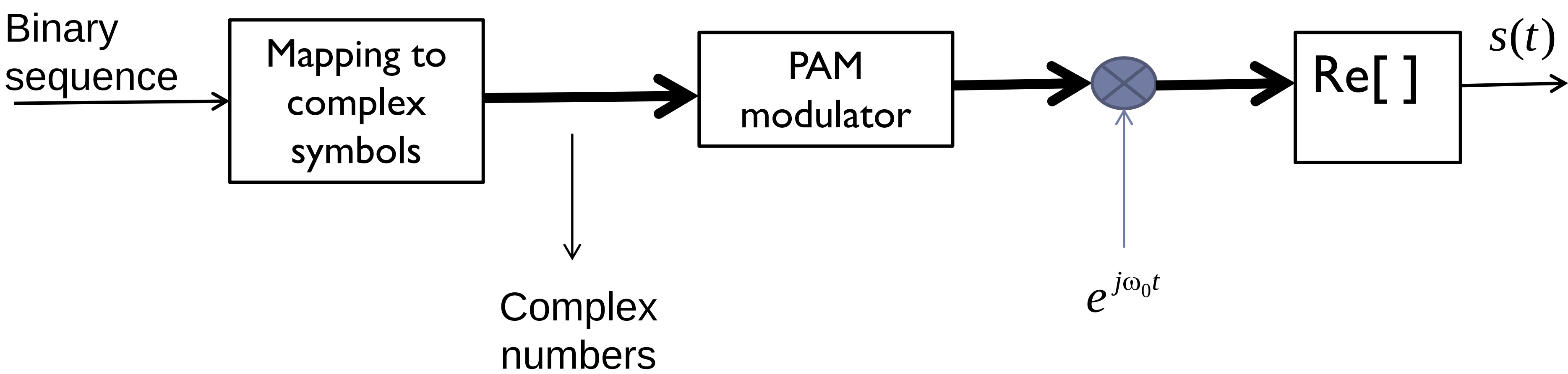
s(t) can be written in terms of its baseband equivalent:

$$s(t) = \mathrm{Re}\left[\sum_k (a_k + jb_k) g_T(t-kT) e^{j\omega_0 t}\right]$$

BANDPASS MODULATION
Quadrature Amplitude Modulation

$$s(t) = \mathrm{Re}\left[\sum_k (a_k + jb_k) g_T(t - kT) e^{j\omega_0 t}\right] \qquad (1)$$

This suggest a possible implementation of the QAM bandpass modulator:



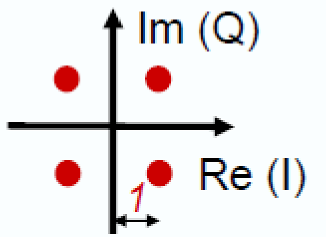
NOTE: any bandpass signal can be expressed with a form as the (1), such as in terms of its baseband components.
Therefore, the above modulator can be used for any bandpass modulation.

When $|c_k| = |a_k + jb_k| = 1$ for any k

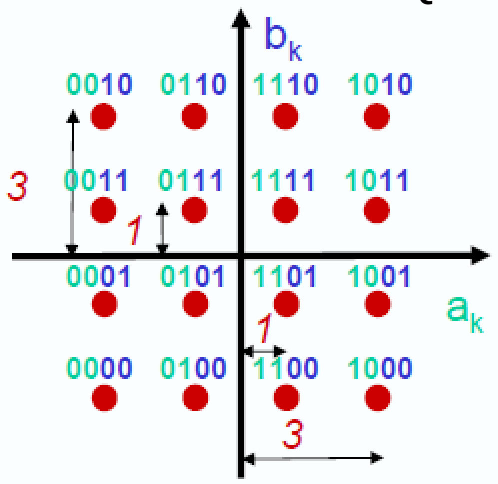
QAM become a PSK modulation.

BANDPASS MODULATION
Quadrature Amplitude Modulation

4-QAM (o anche QPSK)

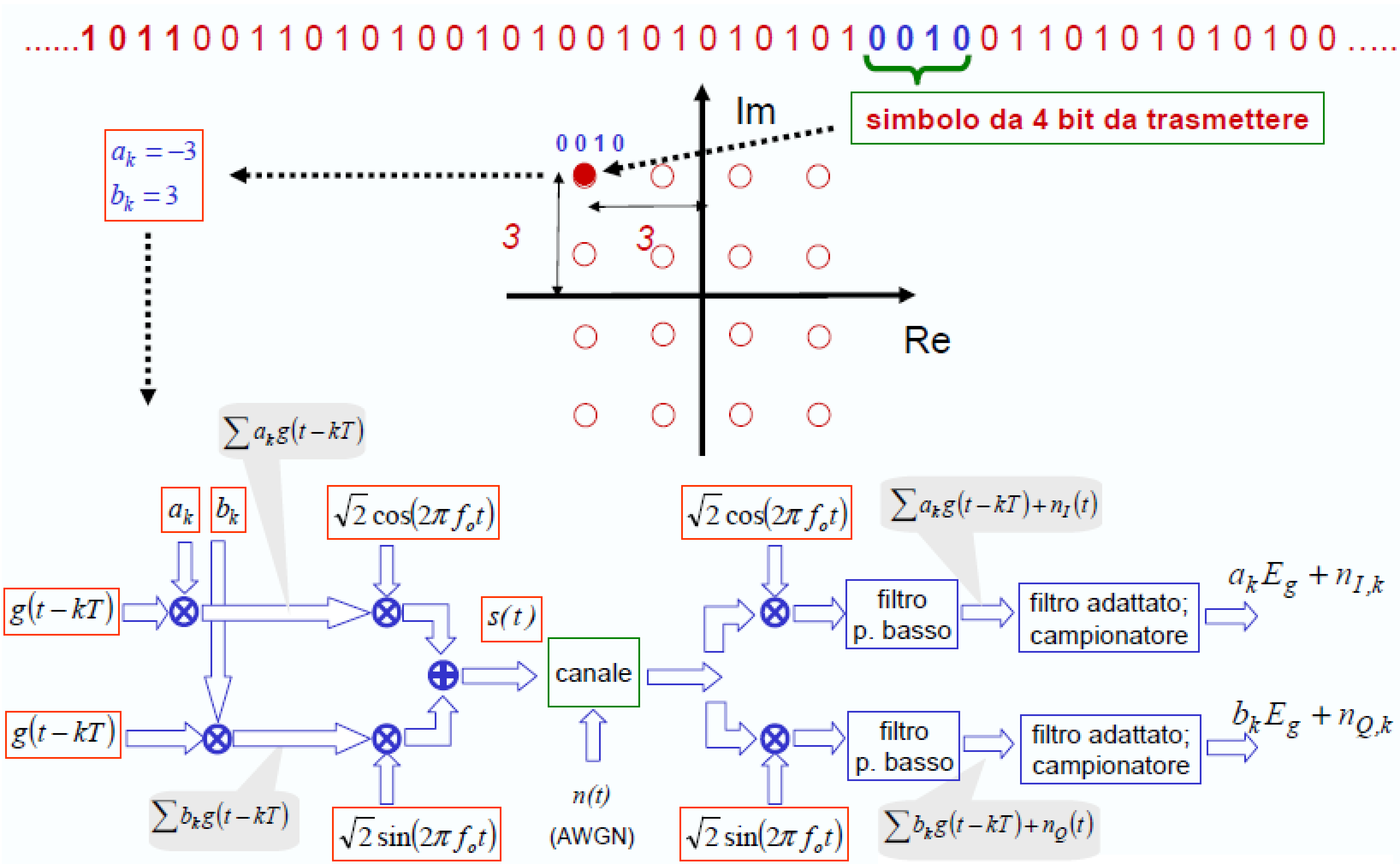


16QAM



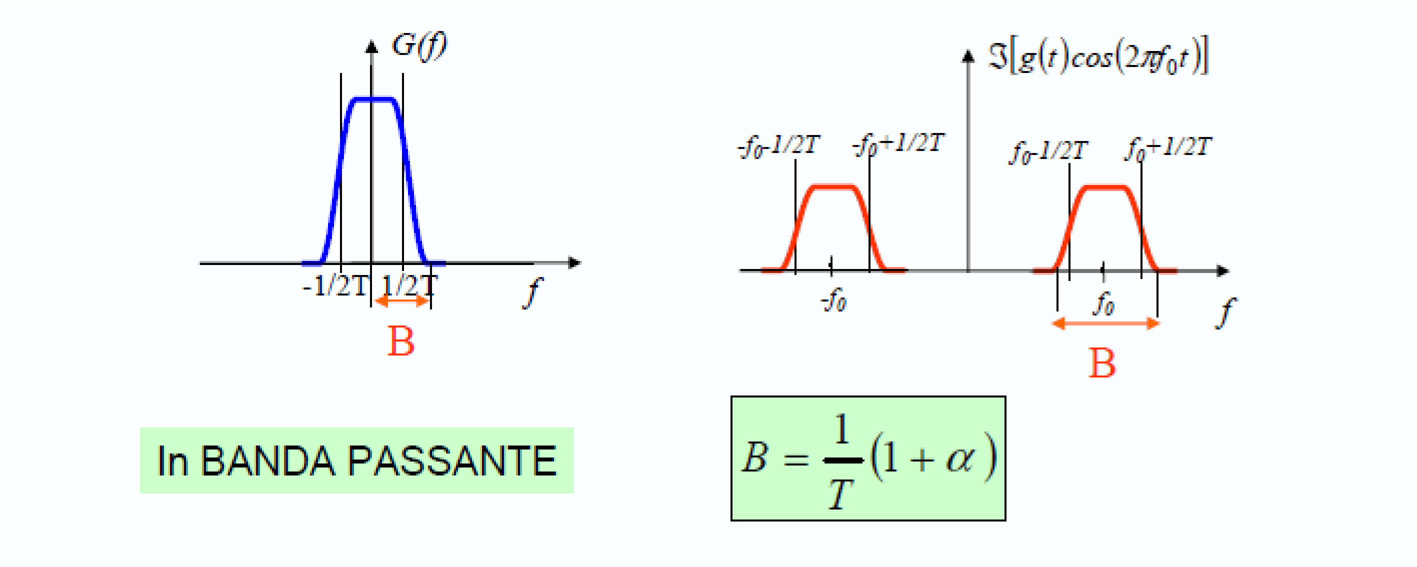
......1 0 1 1 0 0 1 1 0 1 0 1 0 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 1 0 1 0 1 0 1 0 1 0 0 .....

BANDPASS MODULATION
Quadrature Amplitude Modulation



BANDPASS MODULATION
Quadrature Amplitude Modulation

In general, the frequency translated signal occupies a bandwidth which is double wrt the one in baseband.

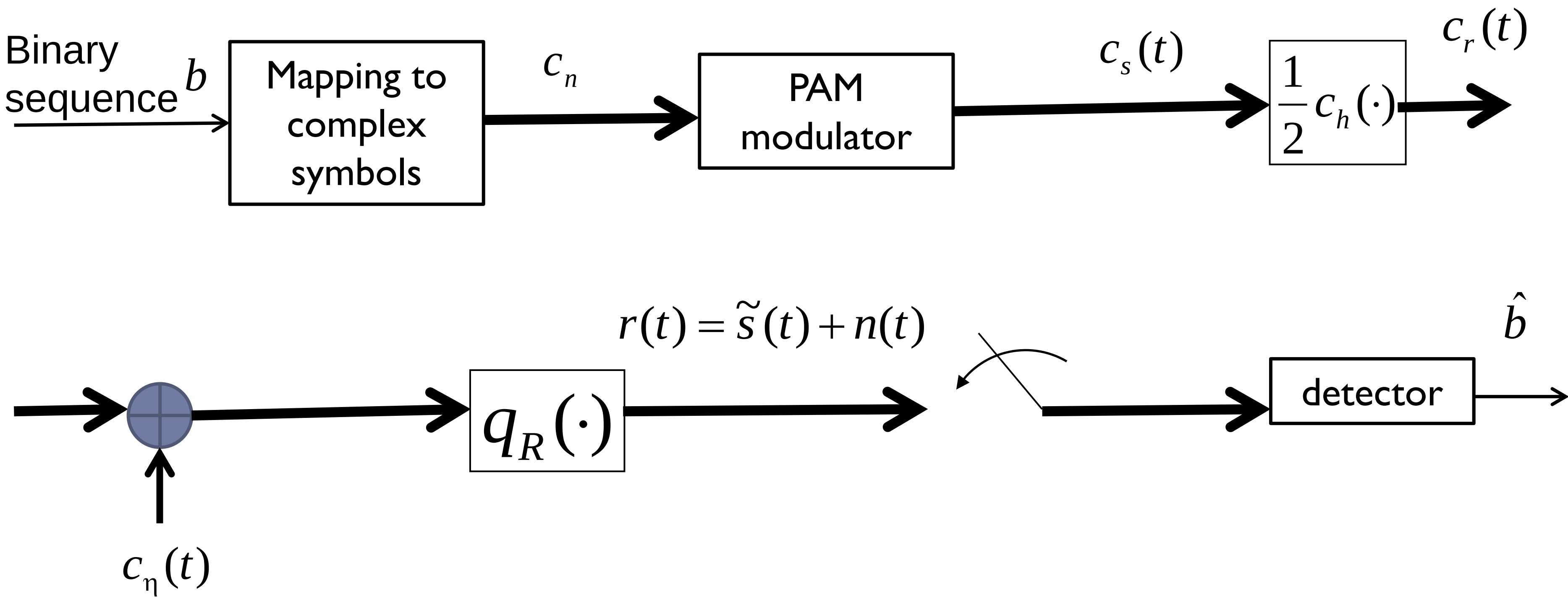


$$B = \frac{1}{T}(1+\alpha)$$

However, if we use two separated channels (phase and quadrature), we are transmitting in parallel two symbols, and hence, the symbol period is double (the baud rate is half) and hence, the minimum bandwidth required according to the Nyquist criteria, is the SAME.

BANDPASS MODULATION
Baseband Equivalent of a bandpass transmitter and receiver

The performance of most DCS will often be described and analyzed as if the transmission channel is a baseband channel using the baseband representation of the bandpass modulator and receiver scheme.



BANDPASS MODULATION
Baseband Equivalent of a bandpass transmitter and receiver

The previous scheme is derived by recalling that:

$$x \longrightarrow \boxed{g(\cdot)} \longrightarrow y \qquad\qquad c_x \longrightarrow \boxed{\frac{1}{2}c_g} \longrightarrow c_y$$

Theorem

A real filter with impulse response g(t) and frequency response G(f) has as baseband equivalent a filter with an impulse response $\frac{1}{2}c_g$ *which is half of the complex envelope of g(t) and as frequency response:*

$$\frac{1}{2}C_g(f)=\frac{1}{2}Z_g(f+f_0)=G_+(f+f_0)$$

BANDPASS MODULATION
Baseband Equivalent of a bandpass transmitter and receiver

IMPORTANT EXAMPLE: simulation of bandpass transmission systems using its baseband equivalent

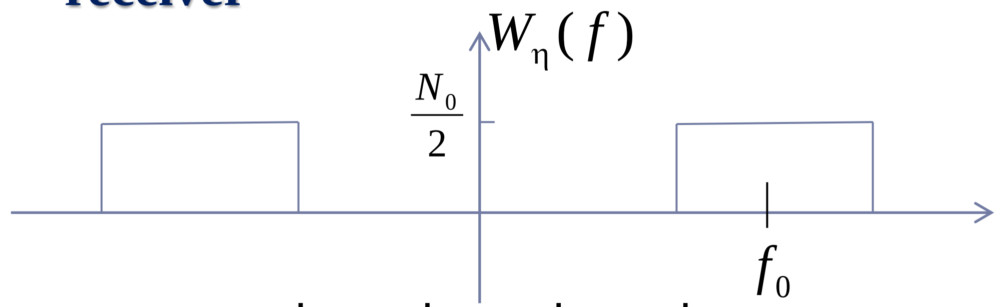
Let us consider a signal modulated with central frequency 1.8 GHz, and bandwidth 2 MHz.
Its maximum frequency content is 1.801 GHz.
If we had to digitally simulate the transmission system, we should sample at frequency:

$$f_{sample} = 2 \cdot f_{max} = 2 \cdot 1.801\,[GHz] \approx 3.6\,\left[G\,\frac{sample}{s}\right]$$

By using the BB equivalent scheme, we should sample at:

$$f_{sample} = 2\,[M\,samples/s]$$

BANDPASS MODULATION
Baseband Equivalent of a bandpass transmitter and receiver



About the noise

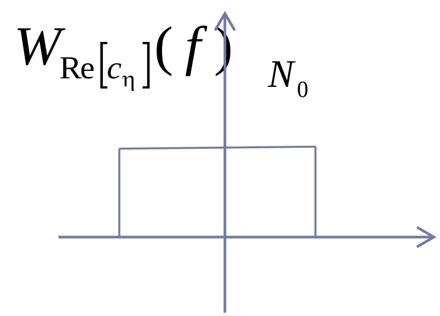
$\eta(t)$ is the bandpass noise

$W_\eta(f)$ is symmetric around the carrier

$$\mathrm{Re}\lfloor c_\eta(t)\rfloor \perp \mathrm{Im}\lfloor c_\eta(t)\rfloor$$

$$W_{\mathrm{Re}[c_\eta]}(f) = W_{\mathrm{Im}[c_\eta]}(f) = W_{c_\eta}(f)/2 = 2W_\eta(f+f_0)$$

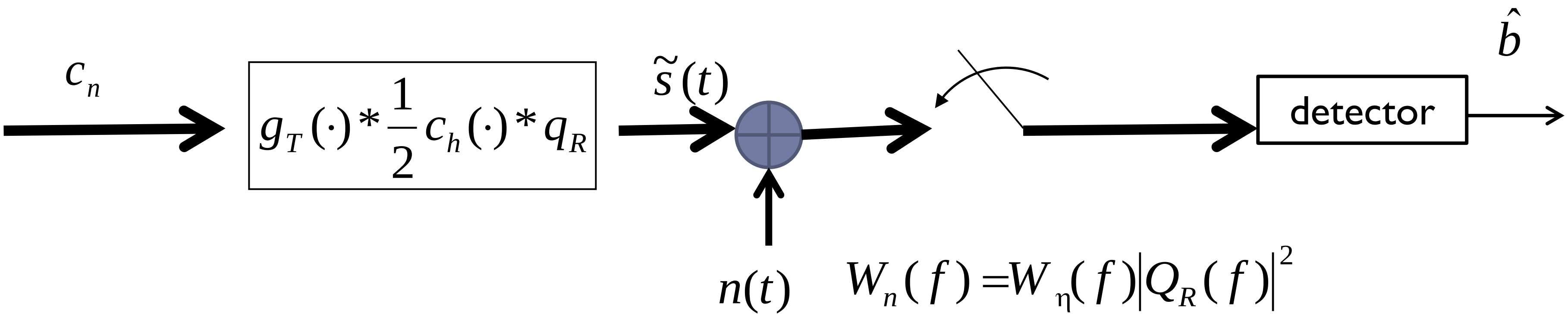
$$W_{c_\eta}(f) = 4W_\eta(f+f_0)$$



The bandpass noise is a complex random process whose real and imaginary parts are statistically independent. The variance of the imaginary and real parts are the same and equal to half of the variance of the complex envelope of the noise.

BANDPASS MODULATION
Baseband Equivalent of a bandpass transmitter and receiver

The previous scheme can be simplified as follows.



$$\boxed{g_A(t) = g_T(\cdot) * \frac{1}{2} c_h(\cdot) * q_R}$$

BANDPASS MODULATION
M-ary FSK

- Signal set:

$$s_m(t) = \sqrt{\frac{2E_s}{T}} \cos\{2\pi(f_c + (m-1)\triangle f)t\} \qquad \begin{array}{l} m = 1, \ldots, M \\ 0 \leq t < T \end{array}$$

where $\triangle f = f_m - f_{m-1}$ with $f_m = f_c + m\triangle f$

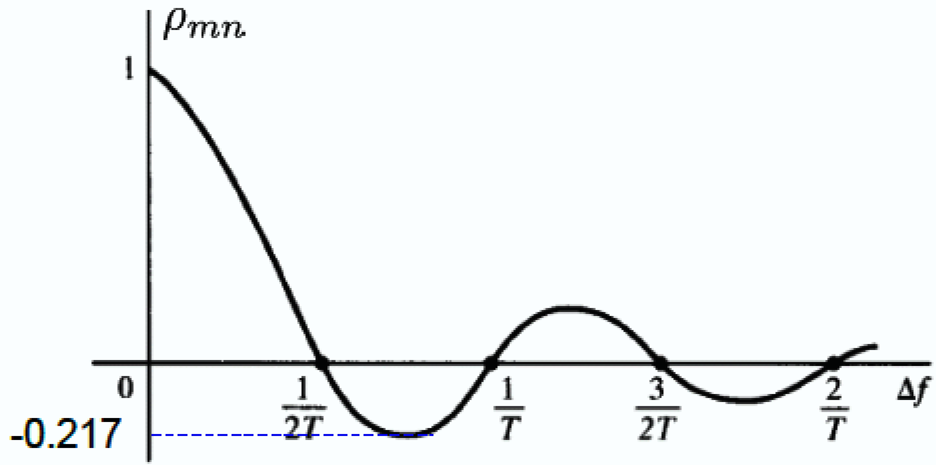
- Correlation between two symbols

$$\rho_{mn} = \frac{1}{E_s} \int_0^T s_m(t) s_n(t) dt$$

$$= \frac{\sin[2\pi(m-n)\triangle f T]}{2\pi(m-n)\triangle f T}$$

$$= \text{sinc}[2(m-n)\triangle f T]$$

BANDPASS MODULATION
M-ary FSK



- For orthogonality, the minimum frequency separation is

$$\Delta f = \frac{1}{2T}$$

BANDPASS MODULATION
M-ary FSK

- M-ary orthogonal FSK has a geometric presenation as M M-dim orthogonal vectors, given as

$$\mathbf{s}_0 = \left(\sqrt{E_s}, 0, 0, \cdots, 0\right)$$

$$\mathbf{s}_1 = \left(0, \sqrt{E_s}, 0, \cdots, 0\right)$$

$$\vdots$$

$$\mathbf{s}_{M-1} = \left(0, 0, \cdots, 0, \sqrt{E_s}\right)$$

- The basis functions are

$$\phi_m = \sqrt{\frac{2}{T}} \cos 2\pi \left(f_c + m\Delta f\right) t$$

BANDPASS MODULATION
M-ary FSK
Demodulation and detection

$$r(t) = \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + 2\pi m \Delta f + \phi_m\right) + n(t)$$

Two methods for the demodulation and detection

Coherent demodulation

All M phases must be estimated

Extremely complex and impractical

Noncoherent demodulation

We will show the principle of non-coherent demodulation of FSK for the binary case

BANDPASS MODULATION
Noncoherent demodulation of FSK

- Since

$$s_1(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t+\theta_1)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t)\cos(\theta_1)-\sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_1 t)\sin(\theta_1)$$

$$s_2(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t+\theta_2)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t)\cos(\theta_2)-\sqrt{\frac{2E_b}{T_b}}\sin(2\pi f_2 t)\sin(\theta_2)$$

- Choose four basis functions as

$$\phi_{1c}(t)=\sqrt{2/T_b}\cos(2\pi f_1 t) \quad \phi_{1s}(t)=-\sqrt{2/T_b}\sin(2\pi f_1 t)$$

$$\phi_{2c}(t)=\sqrt{2/T_b}\cos(2\pi f_2 t) \quad \phi_{2s}(t)=\sqrt{2/T_b}\sin(2\pi f_2 t)$$

- Signal space representation

$$\vec{s}_1=[\ \sqrt{E_b}\cos\theta_1 \quad \sqrt{E_b}\sin\theta_1 \quad 0 \quad 0\ ]$$

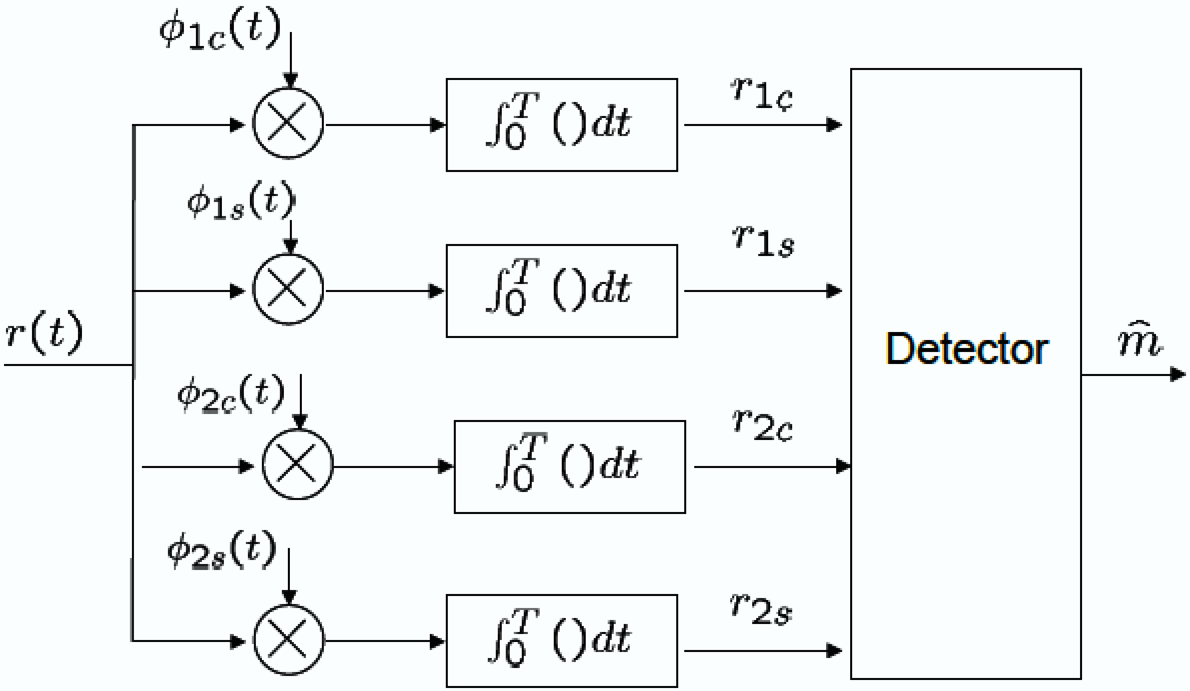
$$\vec{s}_2=[\ 0 \quad 0 \quad \sqrt{E_b}\cos\theta_2 \quad \sqrt{E_b}\sin\theta_2\ ]$$

θ_1, θ_2 are random phases uniformily distributed

Noncoherent demodulation of FSK

- The vector representation of the received signal

$$\vec{r} = [r_{1c} \quad r_{1s} \quad r_{2c} \quad r_{2s}]$$



- ML criterion:

$$f(\vec{r}|\vec{s}_1) \underset{\text{Choose } s_2}{\overset{\text{Choose } s_1}{\gtrless}} f(\vec{r}|\vec{s}_2)$$

- Conditional pdf

$$f(\vec{r}|\vec{s}_1,\theta_1) = \frac{1}{\pi N_0}\exp\left[-\frac{(r_{1c}-\sqrt{E_b}\cos\theta_1)^2+(r_{1s}-\sqrt{E_b}\sin\theta_1)^2}{N_0}\right] \times \frac{1}{\pi N_0}\exp\left[-\frac{r_{2c}^2+r_{2s}^2}{N_0}\right]$$

- Similarly,

$$f(\vec{r}|\vec{s}_2,\theta_2) = \frac{1}{\pi N_0}\exp\left[-\frac{r_{1c}^2+r_{1s}^2}{N_0}\right] \times \frac{1}{\pi N_0}\exp\left[-\frac{(r_{2c}-\sqrt{E_b}\cos\theta_2)^2+(r_{2s}-\sqrt{E_b}\sin\theta_2)^2}{N_0}\right]$$

BANDPASS MODULATION
Noncoherent demodulation of FSK

- ML criterion:

$$f(\vec{r}|\vec{s}_1) \underset{\text{Choose } s_2}{\overset{\text{Choose } s_1}{\gtrless}} f(\vec{r}|\vec{s}_2)$$

- Conditional pdf

$$f(\vec{r}|\vec{s}_1,\theta_1) = \frac{1}{\pi N_0}\exp\left[-\frac{(r_{1c}-\sqrt{E_b}\cos\theta_1)^2+(r_{1s}-\sqrt{E_b}\sin\theta_1)^2}{N_0}\right] \times \frac{1}{\pi N_0}\exp\left[-\frac{r_{2c}^2+r_{2s}^2}{N_0}\right]$$

- Similarly,

$$f(\vec{r}|\vec{s}_2,\theta_2) = \frac{1}{\pi N_0}\exp\left[-\frac{r_{1c}^2+r_{1s}^2}{N_0}\right] \times \frac{1}{\pi N_0}\exp\left[-\frac{(r_{2c}-\sqrt{E_b}\cos\theta_2)^2+(r_{2s}-\sqrt{E_b}\sin\theta_2)^2}{N_0}\right]$$

- For ML decision, we need to evaluate

$$f(\vec{r}|\vec{s}_1) \geq f(\vec{r}|\vec{s}_2)$$

- i.e.

$$\frac{1}{2\pi}\int_0^{2\pi} f(\vec{r}|\vec{s}_1,\theta_1)d\theta_1 \geq \frac{1}{2\pi}\int_0^{2\pi} f(\vec{r}|\vec{s}_2,\theta_2)d\theta_2$$

- Removing the constant terms

$$\left(\frac{1}{\pi N_0}\right)^2 \exp\left[-\frac{{r_{1c}}^2 + {r_{1s}}^2 + {r_{2c}}^2 + {r_{2s}}^2 + E}{N_0}\right]$$

BANDPASS MODULATION
Noncoherent demodulation of FSK

- We have the inequality

$$\frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{2\sqrt{E}r_{1c}\cos(\phi_1)+2\sqrt{E}r_{1s}\sin(\phi_1)}{N_0}\right]d\phi_1$$

$$\geq\frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{2\sqrt{E}r_{2c}\cos(\phi_1)+2\sqrt{E}r_{2s}\sin(\phi_1)}{N_0}\right]d\phi_2$$

- By definition

$$\frac{1}{2\pi}\int_0^{2\pi}\exp\left[\frac{2\sqrt{E}r_{1c}\cos(\phi_1)+2\sqrt{E}r_{1s}\sin(\phi_1)}{N_0}\right]d\phi_1 \;=I_0\left(\frac{2\sqrt{E(r_{1c}{}^2+r_{1s}{}^2)}}{N_0}\right)$$

where $I_0(\cdot)$ is a modified Bessel function of the zeroth order

BANDPASS MODULATION
Noncoherent demodulation of FSK

- Thus, the decision rule becomes: choose s_1 if

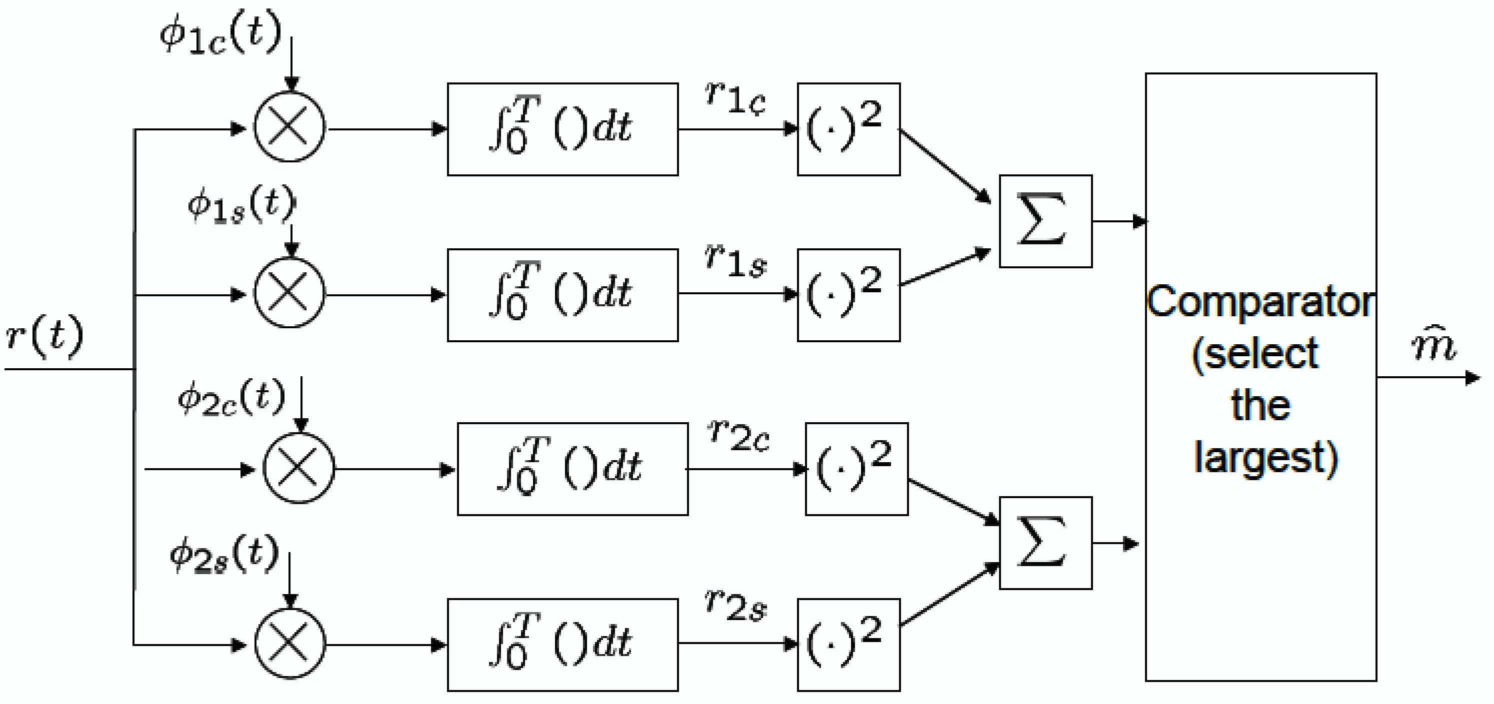
$$I_0\left(\frac{2\sqrt{E(r_{1c}^{\ 2}+r_{1s}^{\ 2})}}{N_0}\right) \geq I_0\left(\frac{2\sqrt{E(r_{2c}^{\ 2}+r_{2s}^{\ 2})}}{N_0}\right)$$

- Noting that this Bessel function is monotonically increasing. Therefore we choose s_1 if

$$\sqrt{r_{1c}^2+r_{1s}^2} \geq \sqrt{r_{2c}^2+r_{2s}^2}$$

- Interpretation: compare the energy in the two frequencies and pick the larger => envelop detector
- Carrier phase is irrelevant in decision making

BANDPASS MODULATION
Noncoherent demodulation of FSK

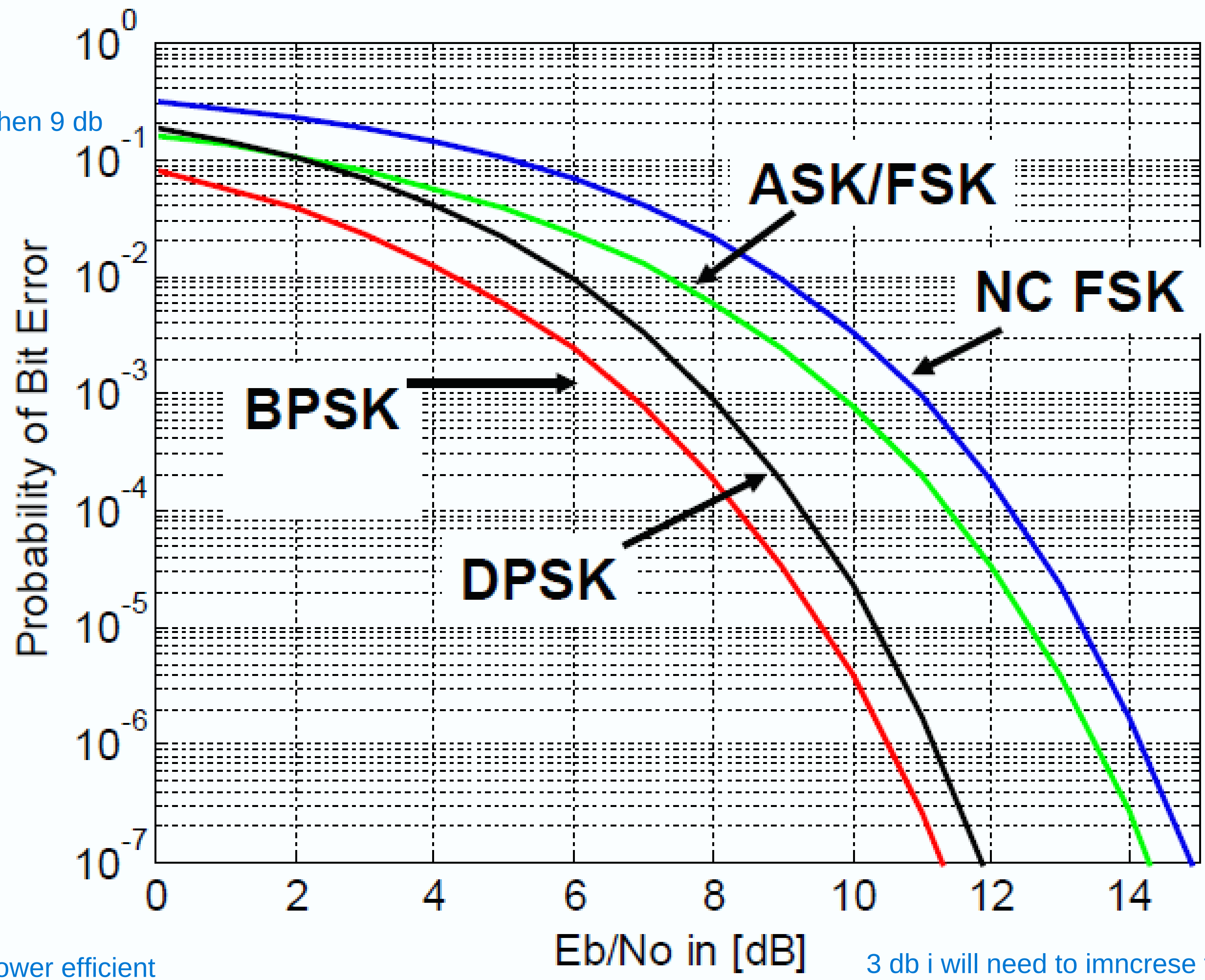


- It can be shown that $P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$

BANDPASS MODULATION
Noncoherent demodulation of FSK

Ber less than 5

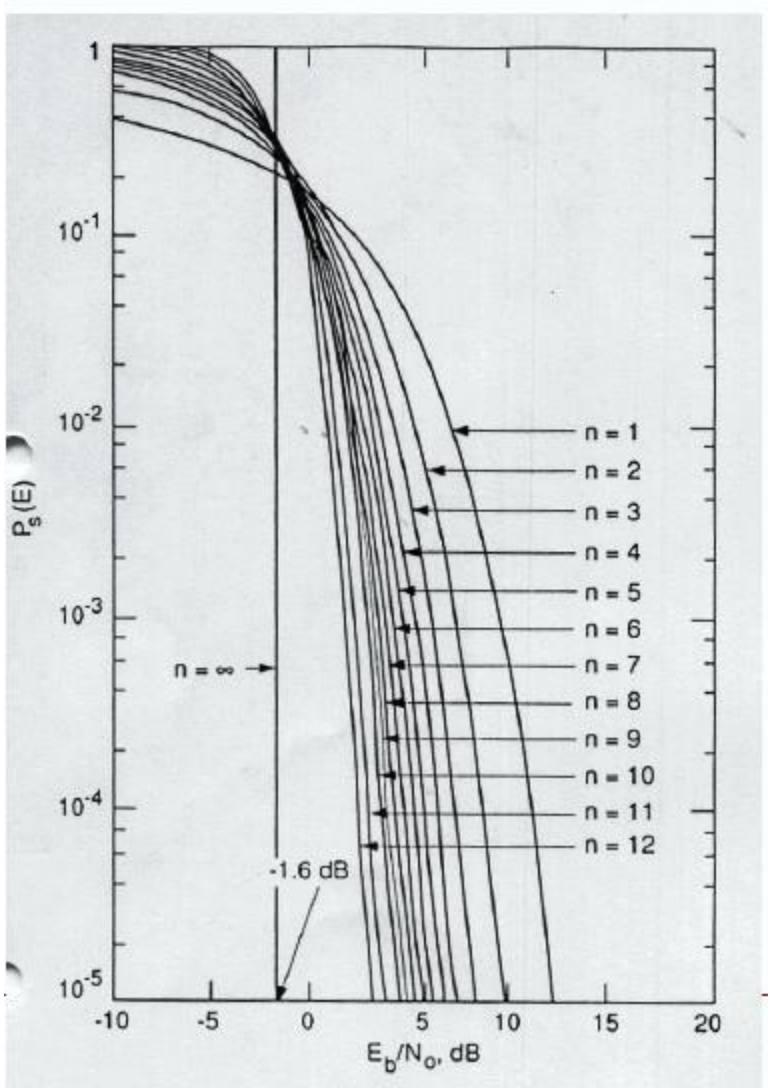
then need srn higher then 9 db



bpsk is more power efficient

3 db i will need to imncrese the power 3 db

BANDPASS MODULATION
Error performance M-FSK



BANDPASS MODULATION
Error performance M-FSK

- When an error occurs anyone of the other symbols may result equally likely.
- Thus, k/2 bits every k bits will on average be in error when there is a symbol error
- Bit error rate is approximately half of the symbol error rate

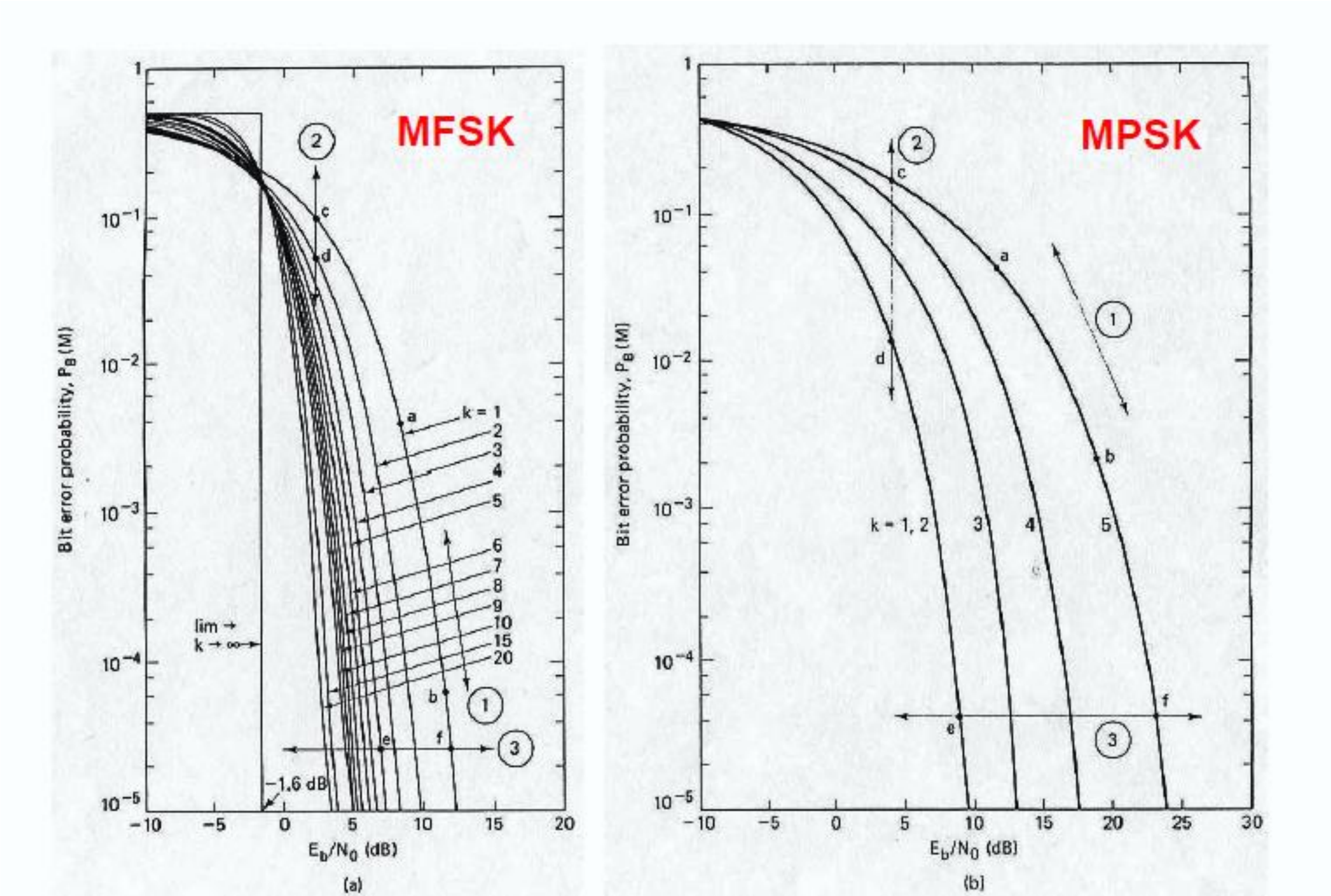
$$P_b \cong \frac{1}{2} P_e$$

Trade-off bandwidth efficiency – power efficiency

- Channel bandwidth and transmit power are two primary communication resources and have to be used as efficient as possible
  - Power utilization efficiency (energy efficiency): measured by the required E_b/N_o to achieve a certain bit error probability
  - Spectrum utilization efficiency (bandwidth efficiency): measured by the achievable data rate per unit bandwidth R_b/B
- It is always desired to maximize bandwidth efficiency at a minimal required Eb/No

BANDPASS MODULATION

Trade-off bandwidth efficiency – power efficiency



Trade-off bandwidth efficiency – power efficiency

- MFSK:
  - At fixed E_b/N_o, increase M can provide an improvement on P_b
  - At fixed P_b increase M can provide a reduction in the E_b/N_o requirement
- MPSK
  - BPSK and QPSK have the same energy efficiency
  - At fixed E_b/N_o, increase M degrades Pb
  - At fixed Pb, increase M increases the Eb/No requirement

MFSK is more energy efficient than MPSK

Trade-off bandwidth efficiency – power efficiency

- In general, bandwidth required to pass MPSK/MQAM signal is approximately given by

$$\boxed{B = \frac{1}{T_s}}$$

- But

$$R_b = \frac{\log_2 M}{T_s} \text{ = bit rate}$$

- Then bandwidth efficiency may be expressed as

$$\boxed{\rho = \frac{R_b}{B} = \log_2 M \ \text{(bits/sec/Hz)}}$$

Comparison

- MFSK:
  - Bandwidth required to transmit MFSK signal is

$$B = \frac{M}{2T}$$

(Adjacent frequencies need to be separated by 1/2T to maintain orthogonality)

  - Bandwidth efficiency of MFSK signal

$$\rho = \frac{R_b}{B} = \frac{2\log_2 M}{M} \quad \text{(bits/s/Hz)}$$

M	2	4	8	16	32	64
ρ (bits/s/Hz)	1	1	0.75	0.5	0.3125	0.1875

As M increases, bandwidth efficiency of MPSK/MQAM increases, but bandwidth efficiency of MFSK decreases.

Trade-off bandwidth efficiency – power efficiency

- To see the ultimate power-bandwidth tradeoff, we need to use Shannon's channel capacity theorem:
  - Channel Capacity is the theoretical upper bound for the maximum rate at which information could be transmitted without error *(Shannon 1948)*
  - For a bandlimited channel corrupted by AWGN, the maximum rate achievable is given by

$$R \leq C = B\log_2(1+SNR) = B\log_2\left(1+\frac{P_s}{N_0 B}\right)$$

- Note that $$\frac{E_b}{N_0} = \frac{P_s T}{N_0} = \frac{P_s}{RN_0} = \frac{P_s B}{RN_0 B} = SNR\frac{B}{R}$$

- Thus $$\frac{E_b}{N_0} = \frac{B}{R}\left(2^{R/B}-1\right)$$

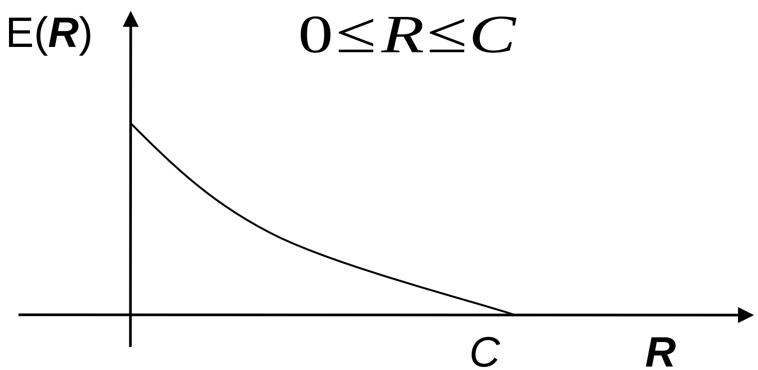
II Shannon Theorem

Given a binary source with Entropy $H_{\infty}(L)$ bit/symbols and a discrete memoryless channel with capacity C, there exist a code with code rate R_c such that:

$$P(e) \le 2^{-L E(R)}, \qquad R = R_c \le C$$

where E(R) is a non-negative decreasing convex function when:

$$0 \le R \le C$$



II Shannon Theorem

Three possible actions to reduce the error probability:

1) to reduce *R* by reducing R_c (**more bandwidth**)

2) to increase the SNR at the receiver by increasing the transmit power (**more power**)

These two approaches to reduce the error probability were applied in the first digital communication systems

They are applicable when the required error probability is not too low and ***without stringent constraints in terms of transmit power and bandwidth***

II Shannon Theorem

Three possible actions to reduce the error probability:

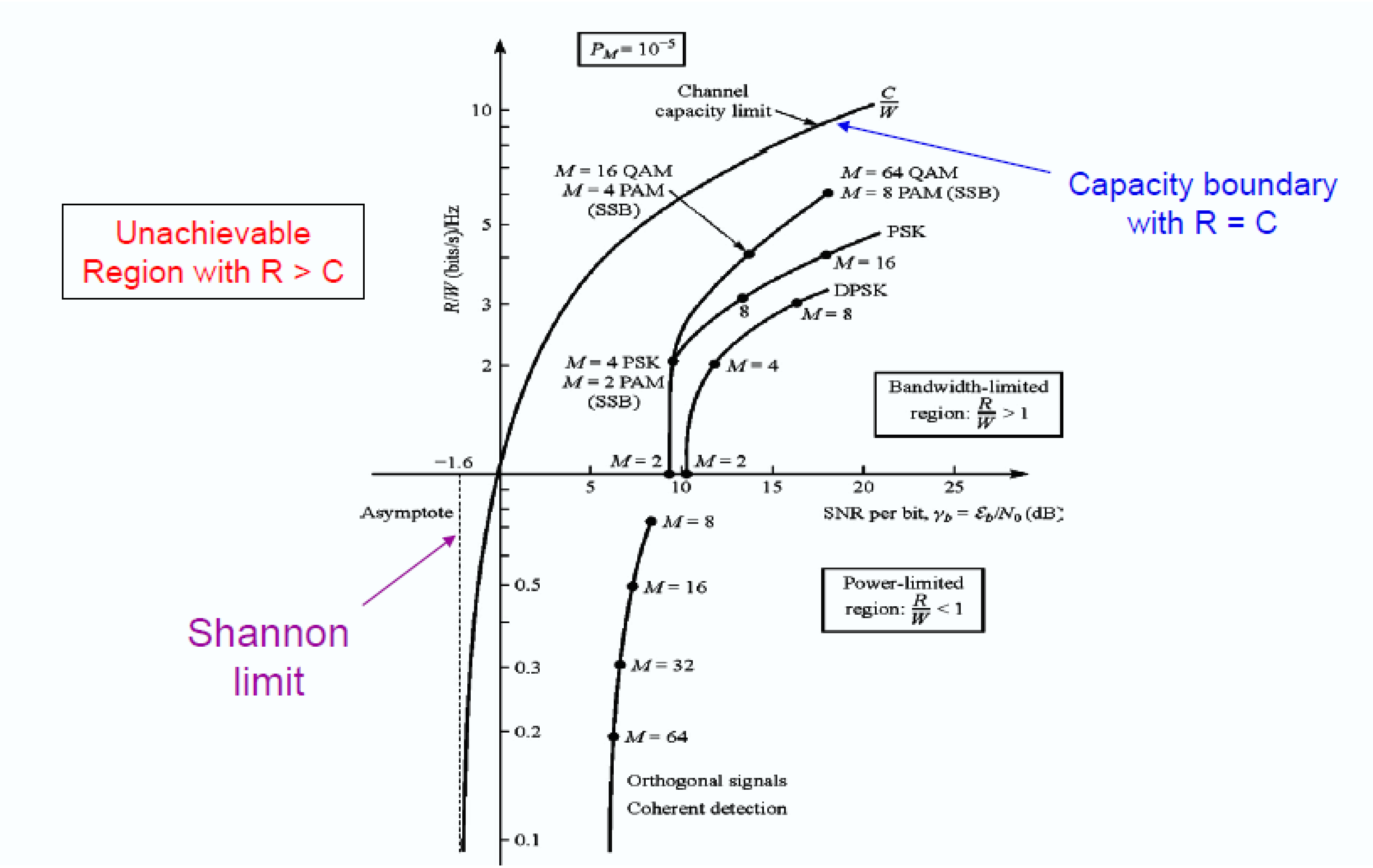
3) to increase *n* while keeping constant R_c

The Shannon theorem states that low error probability can be also achieved by exploiting complex (depending from technological advances) co-dec techniques ***without increasing the required bandwidth or power but increasing the system complexity***

⇓

Channel coding is very important in power limited systems such as satellite systems

BANDPASS MODULATION
Comparison



Comparison

- In the limits as R/B goes to 0, we get

$$\frac{E_b}{N_0} = \ln 2 = 0.693 = -1.59dB$$

  - This value is called the Shannon Limit
  - Received E_b/N_0 must be **>-1.6dB** to ensure reliable communications

- BPSK and QPSK require the same E_b/N_0 of 9.6 dB to achieve $P_e = 10^{-5}$. However, QPSK has a better bandwidth efficiency
- MQAM is superior to MPSK
- MPSK/MQAM increases bandwidth efficiency at the cost of lower energy efficiency
- MFSK trades energy efficiency at reduced bandwidth efficiency.

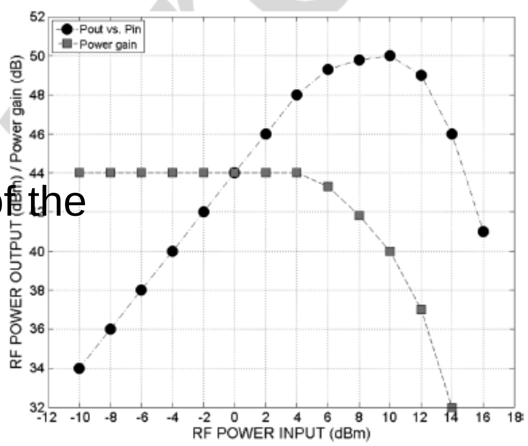
BANDPASS MODULATION
Comparison

In satellite systems, power efficiency is more important than in terrestrial systems

Low order modulations are usually preferable (i.e. 4PSK)

The low spectral efficiency (1 bit/sec/Hz) is usually motivated by reusing the frequency over different satellite beams (multibeam coverage is one of the key element of current broadband satellite systems such as KA-SAT)

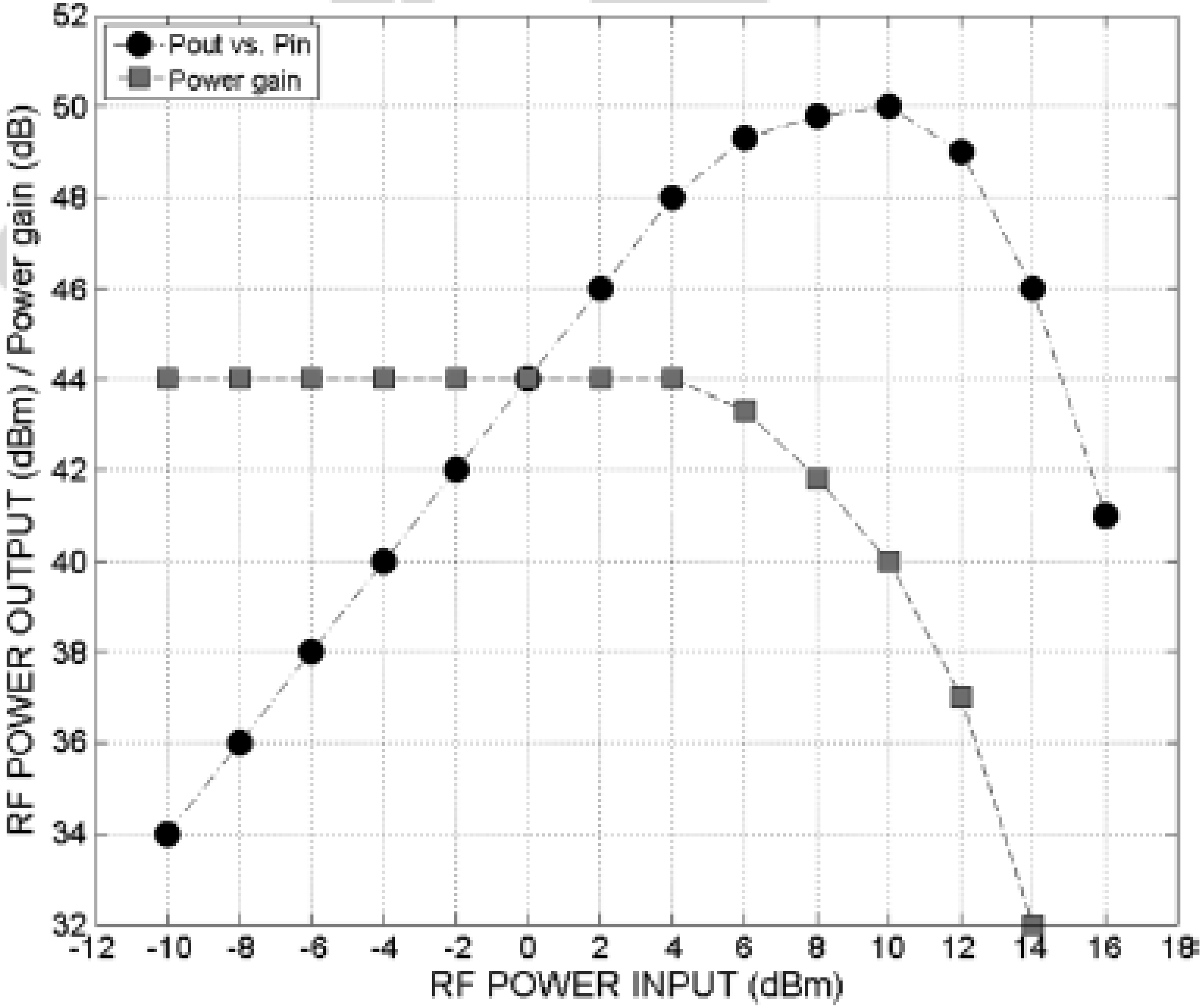
Moreover, since the power on-board is a precious resource, typically the transponder amplifier must work close to the saturation, in the non-linear part of the characteristic curve of the power amplifier. Therefore, the amplifier itself introduce an amplitude distorsion in the signal already at the transmitter,



Constant envelope modulation can work with higher average power levels with the same peak power and hence, it is preferable wrt an amplitude modulation (i.e. M-PSK instead of QAM)

BANDPASS MODULATION

Comparison

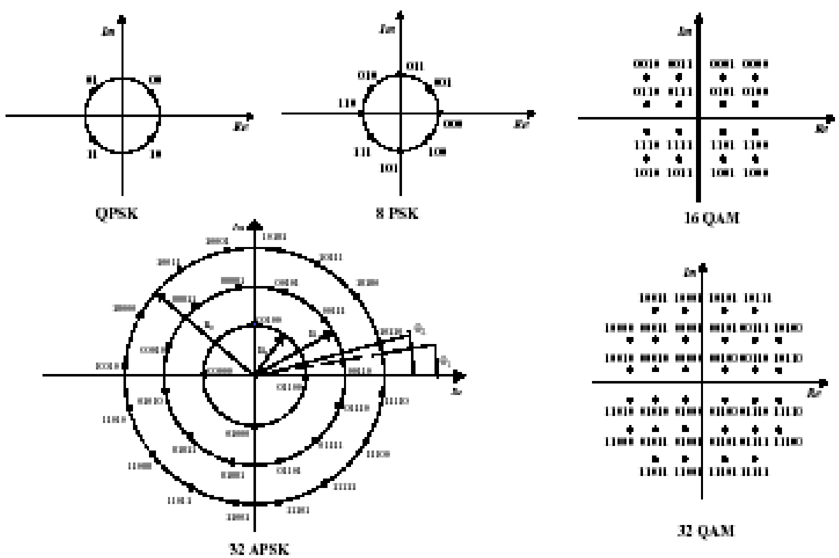


Comparison

FSK and PSK are constant envelope modulations which are desirable in some applications where amplitude is very much distorted like in satellite systems with power amplifiers working in the non linear regions

FSK is used in applications when bandwidth is less precious than power (again, in satellite systems)

Standard DVB-S2 foreseen the use of different modulations



BANDPASS MODULATION

Comparison

- BPSK:
  - WLAN IEEE802.11b (1 Mbps)
- QPSK:
  - WLAN IEEE802.11b (2 Mbps, 5.5 Mbps, 11 Mbps)
  - 3G WDMA
  - DVB-T (with OFDM)
- QAM
  - Telephone modem (16QAM)
  - Downstream of Cable modem (64QAM, 256QAM)
  - WLAN IEEE802.11a/g (16QAM for 24Mbps, 36Mbps; 64QAM for 38Mbps and 54 Mbps)
  - LTE Cellular Systems
- FSK:
  - Cordless telephone

BANDPASS MODULATION
Comparison

Bluetooth: GFSK *Gaussian shaped Frequency Shift Keying.*

Zigbee

PHY	Frequency Band	Channels	parameters		parameters		
			Chip rate	Modulation	Bit rate	Symbol rate	From bits to symbols
800/915 MHz	868-870 MHz	0	300 kchip/s	BPSK	20 kb/s	20 kbaud	Binary
	902- 928 MHz	From 1 to 10	600 kchip/s	BPSK	40 kb/s	40 kbaud	Binary
2.4 GHz	2.4-2.4835 GHz	From 11 to 26	2.0 Mchip/s	O-QPSK	**250 kb/s**	62.5 kbaud	16-ary Orthogonal

BANDPASS MODULATION
Comparison

802.15.6 – Narrowband PHY

Band (MHz)	Number of Channels	Modulation	Symbol Rate (ksps)	Code Rate (k/n)	Spreading Factor (S)	Pulse Shape	Information Data Rate (kbps)	Support
402 – 405	10	π/2-DBPSK	187.5	51/63	2	SRRC	75.9	Mandatory
					1		151.8	
		π/4-DQPSK					303.6	
		π/8-D8PSK					455.4	Optional
863 – 870 **902 – 928** **950 – 956**	14 60 16	π/2-DBPSK	250	51/63	2	SRRC	101.2	Mandatory
					1		202.4	
		π/4-DQPSK					404.8	
		π/8-D8PSK					607.1	Optional
2360 – 2400 **2400 – 2483.5**	39 79	π/2-DBPSK	600	51/63	4	SRRC	121.4	Mandatory
					2		242.9	
					1		485.7	
		π/4-DQPSK					971.4	

Telephone modem

TABLE 9.5 Evolution of Dial-Line Telephone Modem Standards

Year	Name	Maximum Bit Rate (bits/s)	Signaling Rate (symbols/s)	Modulation Technique	Signaling Efficiency (bits/symbol)
1984	V.32	9600	2400	2-D Trellis Coded 32-QAM	4
1991	V.32bis	14,400	2400	2-D Trellis Coded 128-QAM	6
1994	V.34	28,800	2400, 2743, 2800, 3000, 3200, 3429	4-D Trellis Coded 960-QAM	≈ 9
1996	V.34	33,600	2400, 2743, 2800, 3000, 3200, 3429	4-D Trellis Coded 1664-QAM	≈ 10
1998	4.90	downstream: 56,000 upstream: 33,600	8000 as in V.34	PCM* (*M*-PAM) as in V.34	7 ≈ 10
2000	V.92	downstream: 56,000 upstream: 48,000	8000 8000	PCM* (*M*-PAM) Trellis Coded PCM*	7 6

*In the G.711 ITU-T Recommendation, PCM is the term used for *M*-ary PAM signaling.

High SNR (around 30dB)
Small bandwidth: 3kHz