Digital Communications
Pulse Modulation
transition from analog to digital modulation

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DIGITAL COMUNICATION SYSTEM
Pulse modulation

Whatever is the source of information, in a DCS the message is converted in a sequence of digits
This sequence of digits is called **baseband signal**

The channel through which it is possible to send baseband signals are called **baseband channels** (pair of wires, coaxial cable)

No channel can be used for the transmission of binary digits without first transforming the digits to waveforms that are compatible with the channel.

For baseband channel, compatible waveforms are pulses

Pulse modulation

In continous wave modulation, some parameters of a sinusoidal carrier wave is varied continously according with the message signal

In case of pulse modulation some parameters of a pulse train is varied in accordance with the message signal (i.e. the digits being sent).

Pulse modulation waveforms

There are three ways to modulate information on to a sequence of pulses:

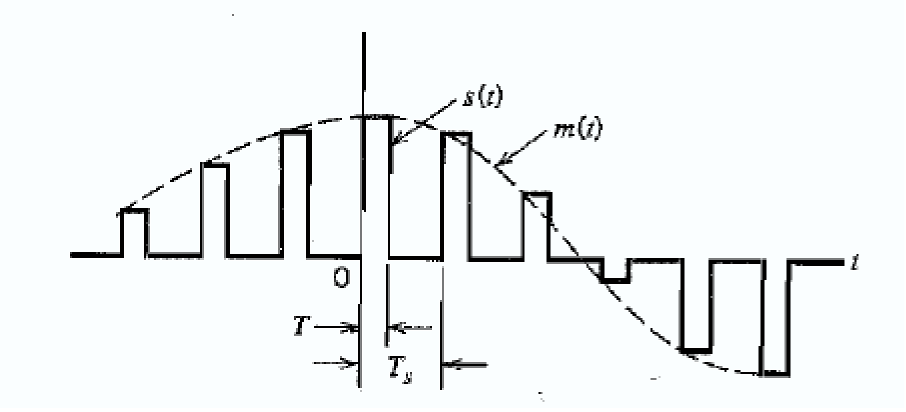
•PAM (pulse amplitue modulation)
•PDM (pulse duration modulation)
•PPM (pulse position modulation)

❑When the information samples are not quantizied, we have **analog pulse modulation**

❑ When they are quantized, we have **digital pulse modulation**

DIGITAL COMUNICATION SYSTEM
Analog Pulse modulation

Analog PAM



The amplitude of regularly spaced pulses are varied in proportion to the corresponding sample values of a continous message signal

<u>Pulses could be rectangular but also of other shapes</u>

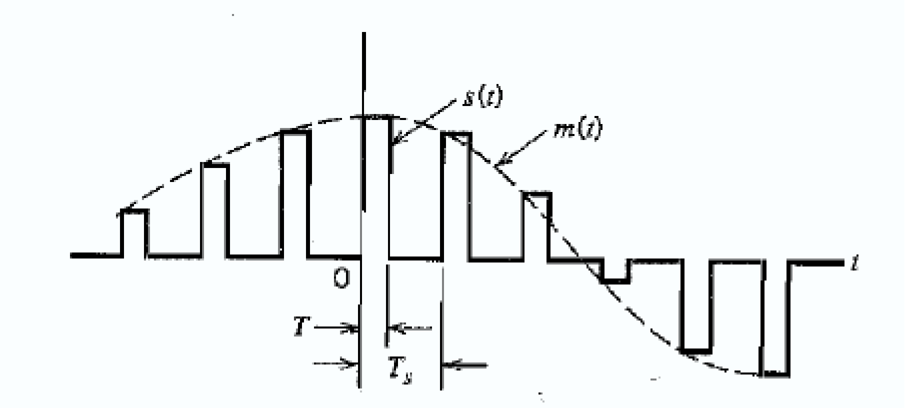
In case of rectangular pulses, the PAM signal is generated by the following two operations:

- **Instantaneous sampling** of the message signal m(t) with sampling period Ts and sampling frequency fs=1/Ts which verify the Nyquist theorem

- **Lengthening** the duration of each sample up to a duration T

DIGITAL COMUNICATION SYSTEM
Analog Pulse modulation

Analog PAM

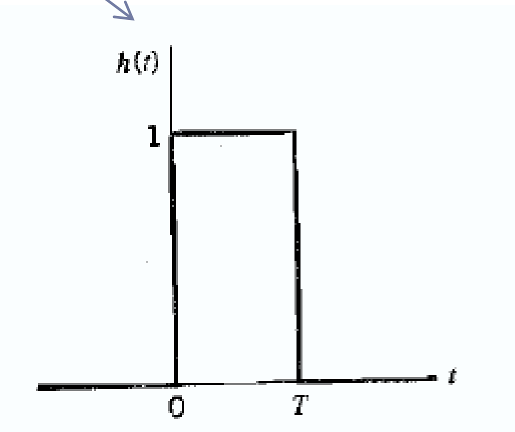


These two operations are commonly jointly referred as "sample and hold"

PAM signal

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_S)h(t-nT_S) = m_\delta * h(t)$$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_S)\delta(t-nT_S)$$



Analog PAM

Proof

$$m_\delta(t) * h(t) = \int_{-\infty}^{\infty} m_\delta(\tau) h(\tau - nT_s) d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s) \delta(\tau - nT_s) h(\tau - nT_s) d\tau$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s) h(\tau - nT_s) d\tau$$

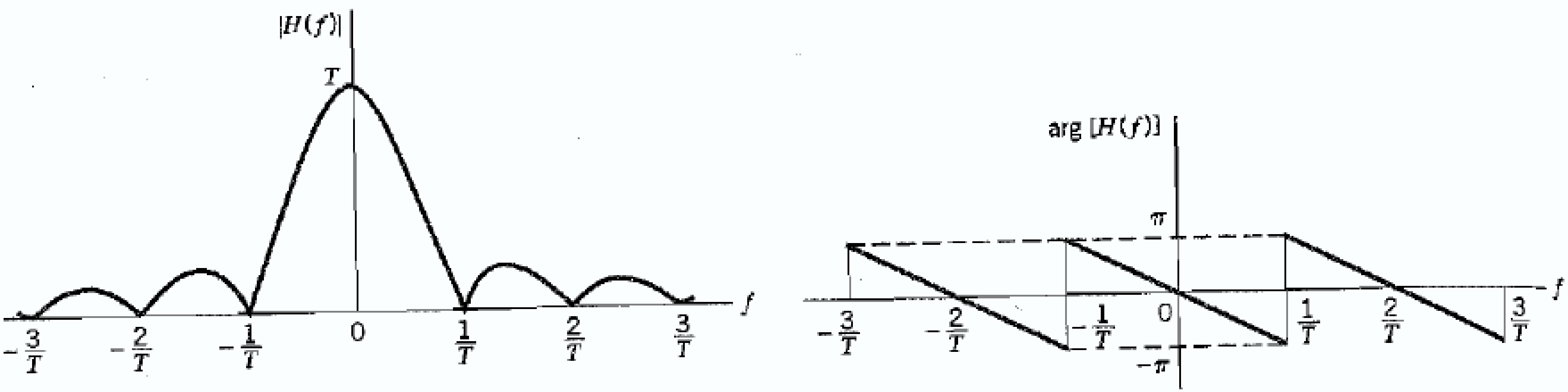
$$= \sum_{n=-\infty}^{\infty} m(nT_s) h(\tau - nT_s) = s(t)$$

Analog PAM

$$S(f) = M_\delta(f)H(f)$$

$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)$$

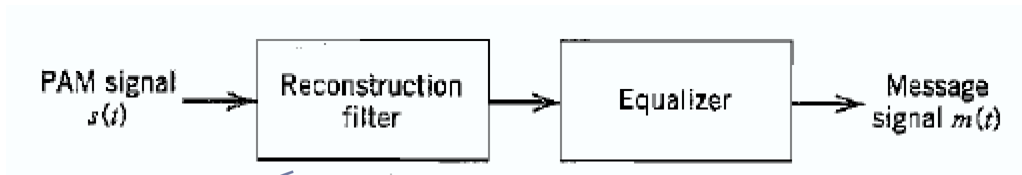
$$H(f) = T \sin c(fT)e^{-j\pi fT}$$



(b)

Analog PAM

Reconstruction of m(t)



the interpolating filter of the sampling theorem

the output of not just $M(f)$ but

$$M(f)H(f)$$

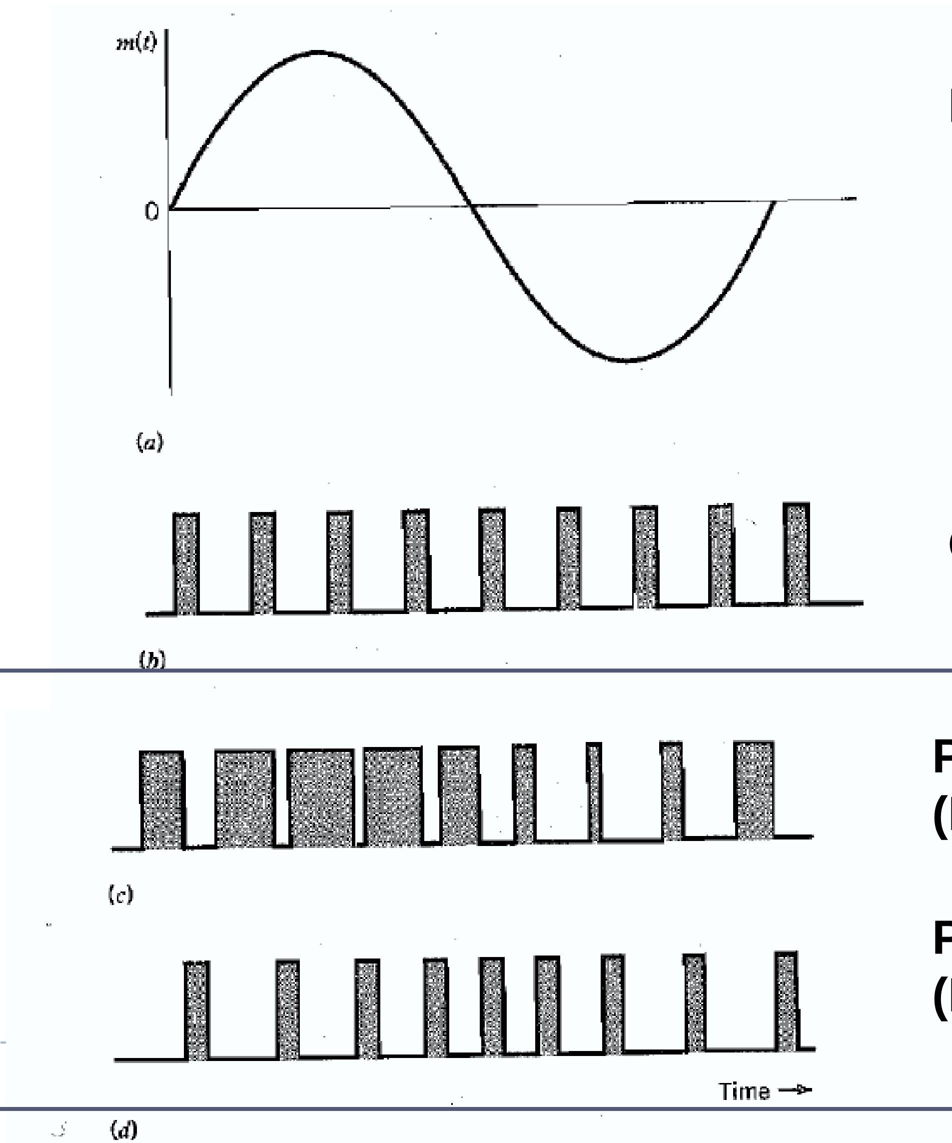
the signal is recovered with a distorsion in amplitude and it is delayed of T/2

Usually the distorsion is neglectable
When it is not, an equalizer is needed with ideally the following magnitude response

$$\frac{1}{|H(f)|} = \frac{\pi}{\sin \pi f T}$$

DIGITAL COMUNICATION SYSTEM
Analog Pulse modulation

Other types of analog pulse modulation



Modulating signal

Carrier (train of pulses)

Pulse Duration Modulation (PDM)

Pulse Position Modulation (PPM)

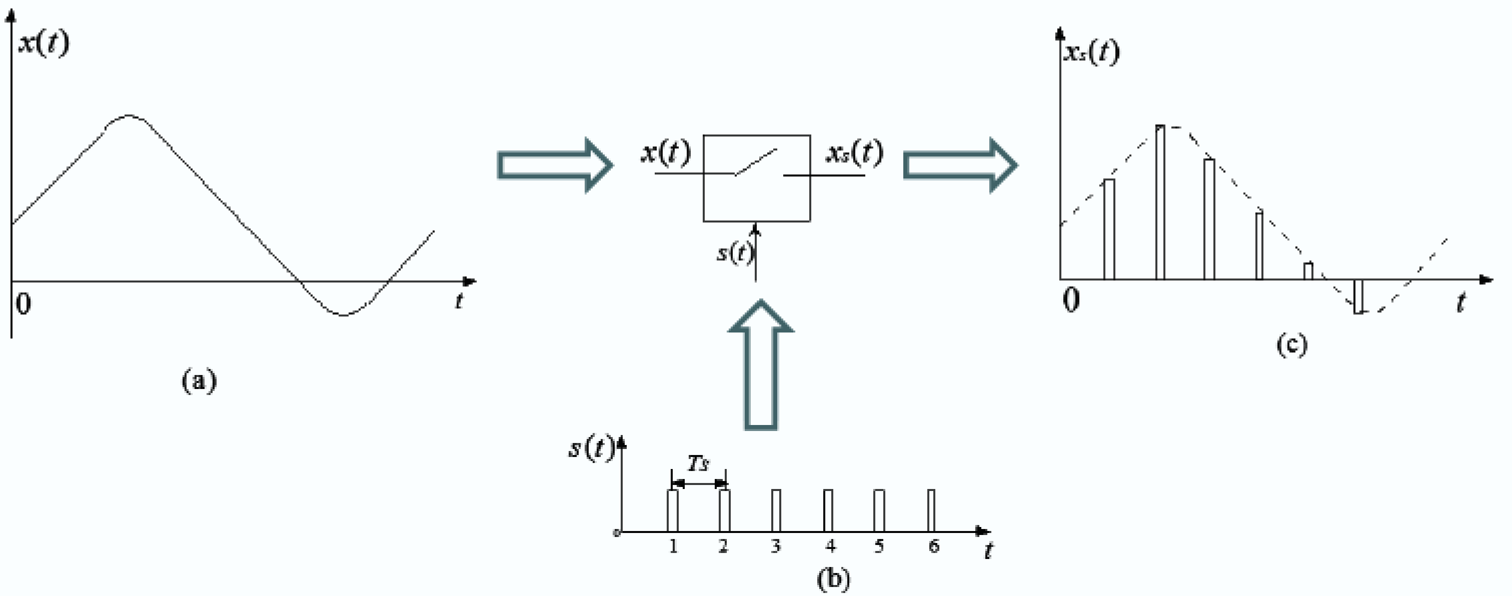
Sampling Process

It is the basic operation of any digital signal processing and digital communications.
It is the process that converts an analog signal into a sequence of samples that are usually spaced uniformly in time.
The utility of doing this operations stands from the fact that by properly choosing the sampling rate, it is possible to recover the original signal from the samples (Sampling Theorem).

Let us consider an arbitrary signal x(t) of finite energy, which is specified for all time.

DIGITAL COMUNICATION SYSTEM
Pulse modulation

Sampling Process



$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) = x(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

DIGITAL COMUNICATION SYSTEM
Pulse modulation

$$X_\delta(f) = X(f) * \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_s}\right) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(f - \frac{n}{T_s}\right)$$

T_s

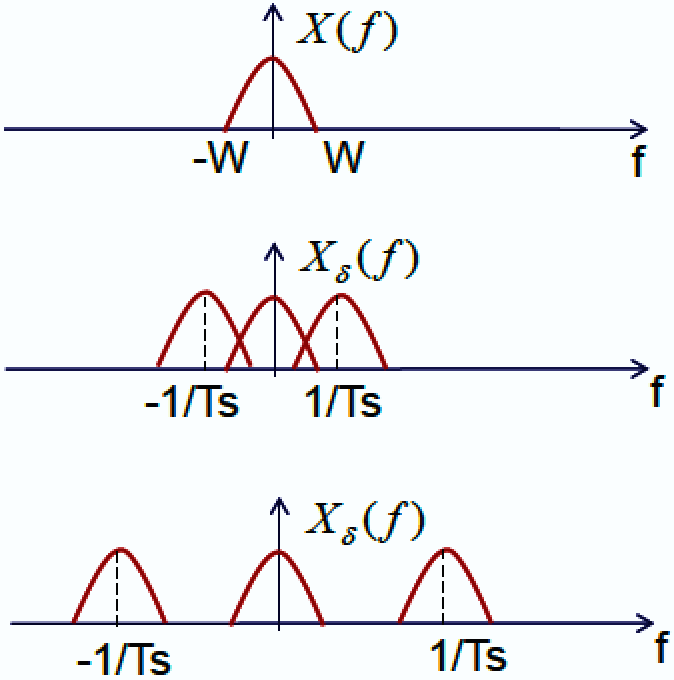
f

- If $f_s = \frac{1}{T_s} < 2W$ or $T_s > \frac{1}{2W}$

  aliasing error, reconstruction is not possible,

- The minimum sampling rate is known as **Nyquist sampling rate**

  $$f_s = 2W$$



DIGITAL COMUNICATION SYSTEM
Pulse modulation

$$X_\delta(f) = X(f) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_s}\right) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}X\left(f - \frac{n}{T_s}\right)$$

T_s

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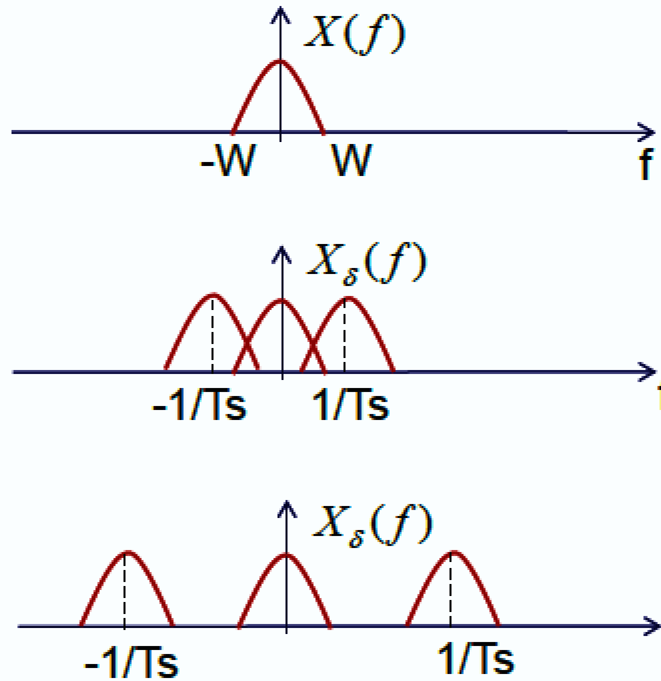


- LPF with frequency response

T_s

$$H(f) = \begin{cases} T_s & |f| < W \\ 0 & |f| \geq \dfrac{1}{T_s} - W \end{cases}$$

- Ideal LPF

$$H(f) = T_s \Pi\left(\frac{f}{2W'}\right) \Leftrightarrow h(t) = 2W'T_s \mathrm{sinc}(2W't)$$

where $W \leq W' < \frac{1}{T_s} - W$

- With this choice, we have

$$
\begin{aligned}
x(t) &= x_\delta(t) * 2W'T_s \operatorname{sinc}(2W't) \\
&= \left(\sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \right) * 2W'T_s \operatorname{sinc}(2W't) \\
&= \sum_{n=-\infty}^{\infty} 2W'T_s x(nT_s) \operatorname{sinc}\left[2W'(t - nT_s) \right]
\end{aligned}
$$

DIGITAL COMUNICATION SYSTEM
Pulse modulation
OVERSAMPLING

<u>Oversamplig:</u> sampling at a sampling rate which is n-times (with n usually equal to 4 or more) the Nyquist sampling rate

Oversampling is the most economic solution for the task of transforming an analog signal to a digital signal or the reverse.

This is due to the fact that signal processing performed with high performance analog equipment is typically much more costly than using digital signal processing equipment.

<u>A/D conversion without oversampling:</u>
1) Signal passes through a high performance analog low pass filter to limit its bandwidth (condition of the Nyquist theorem)
2) The filtered signal is sampled at the Nyquist rate for the *approximated* bandlimited signal.
3) The samples are processed by an A/D converter that maps the continuous valued samples to a finite list of discrete output levels.

DIGITAL COMUNICATION SYSTEM
Pulse modulation
OVERSAMPLING

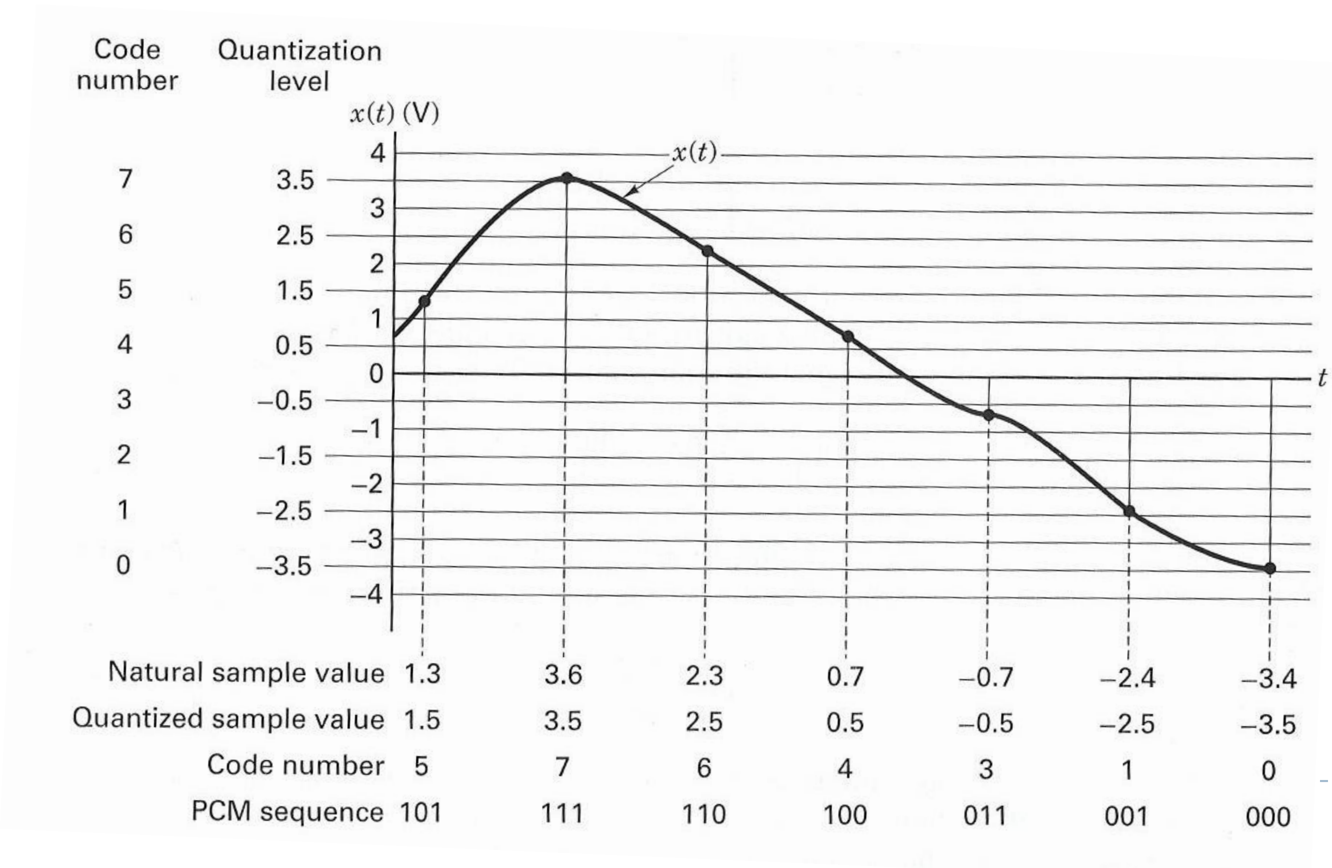
<u>A/D conversion **with** oversampling:</u>

1) Signal passes through a low performance analog low pass filter to limit its bandwidth
2) The pre-filtered signal is sampled at the rate higher than the Nyquist rate
3) The samples are processed by an A/D converter that maps the continuous valued samples to a finite list of discrete output levels.
4) Digital samples are then processed by a high performance digital filter to reduce the bandwidth of the digital samples
5) The sample rate at the output of the digital filter is reduced in proportion to the bandwidth reduction obtained by this digital filter.

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation

In case of digital pulse modulation, the train of pulses is modulated by a sequence of digital symbols (belonging to a finite alphabet).

The analog signal is sampled, quantized, and each quantized symbol is associated to a "digital word" (encoding process) which is used to modulate the train of pulses.



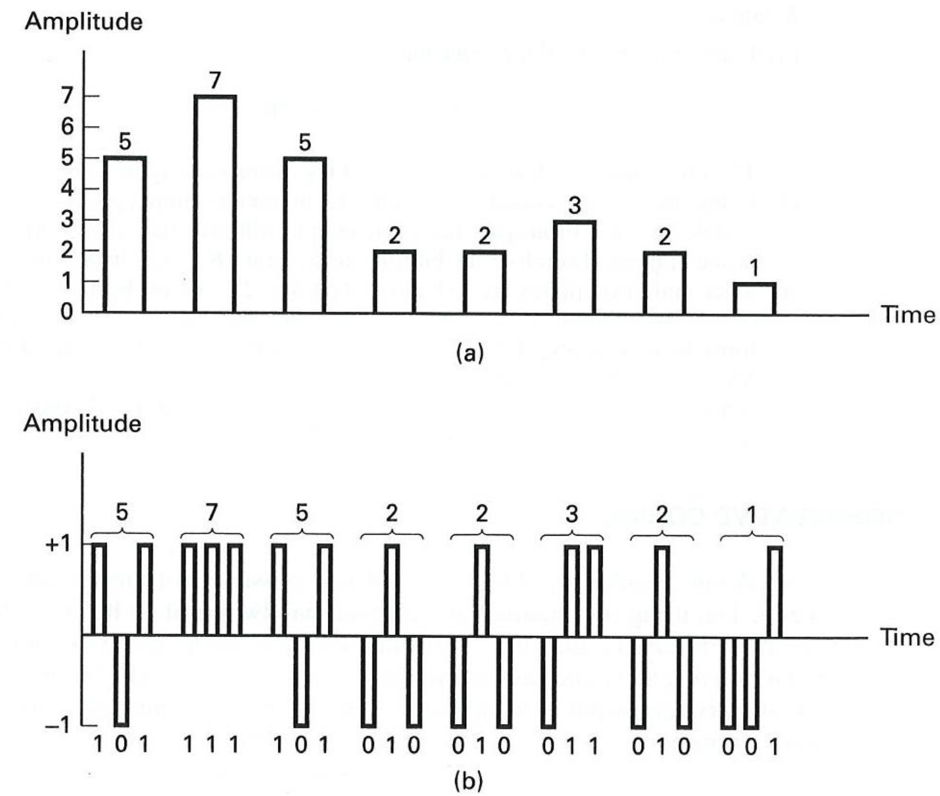
DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation

When pulse modulation is applied to a binary symbol, the resulting binary waveform is called **Pulse Code Modulation (PCM)** waveforms.

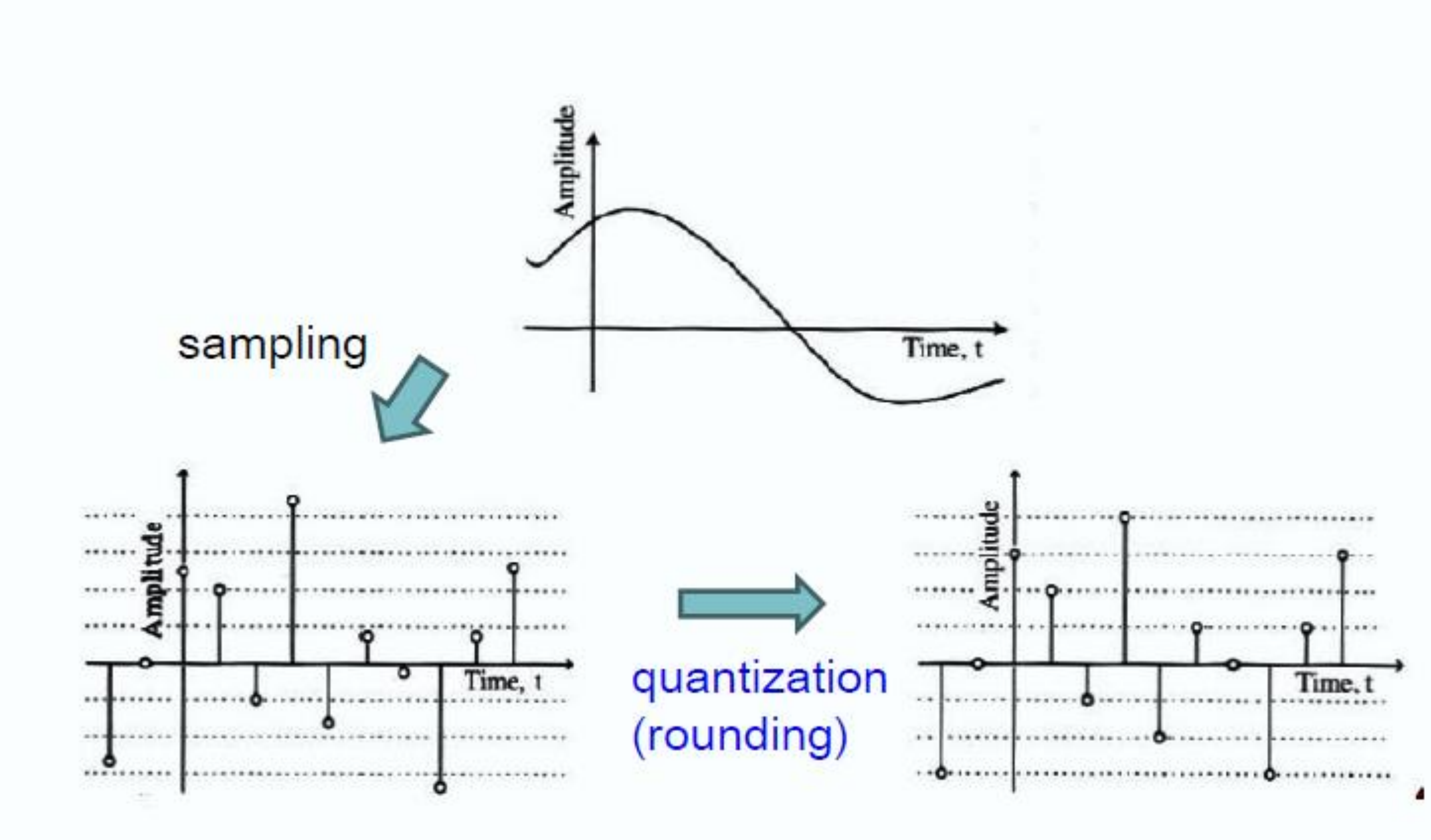
When pulse modulation is applied to a non binary symbol, the resulting waveform is called M-ary (M is the cardinality of the alphabet of digital words) pulse waveform.

M-ary waveform

PCM waveform



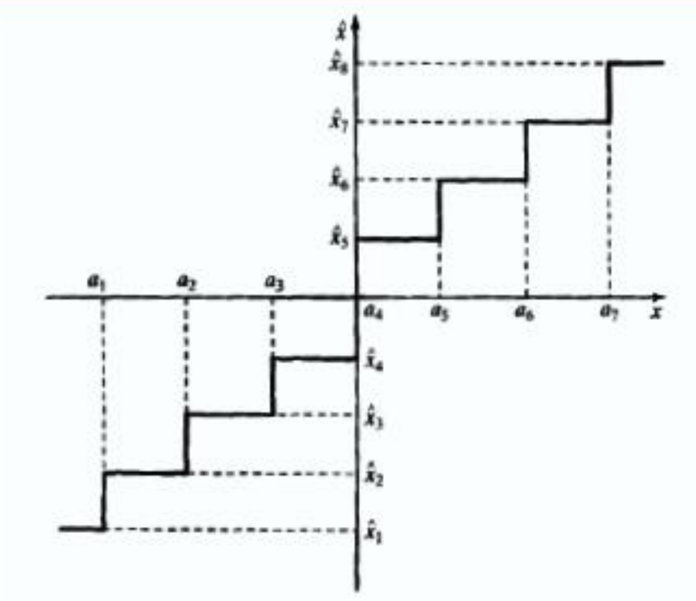
Quantization Process



Quantization Process

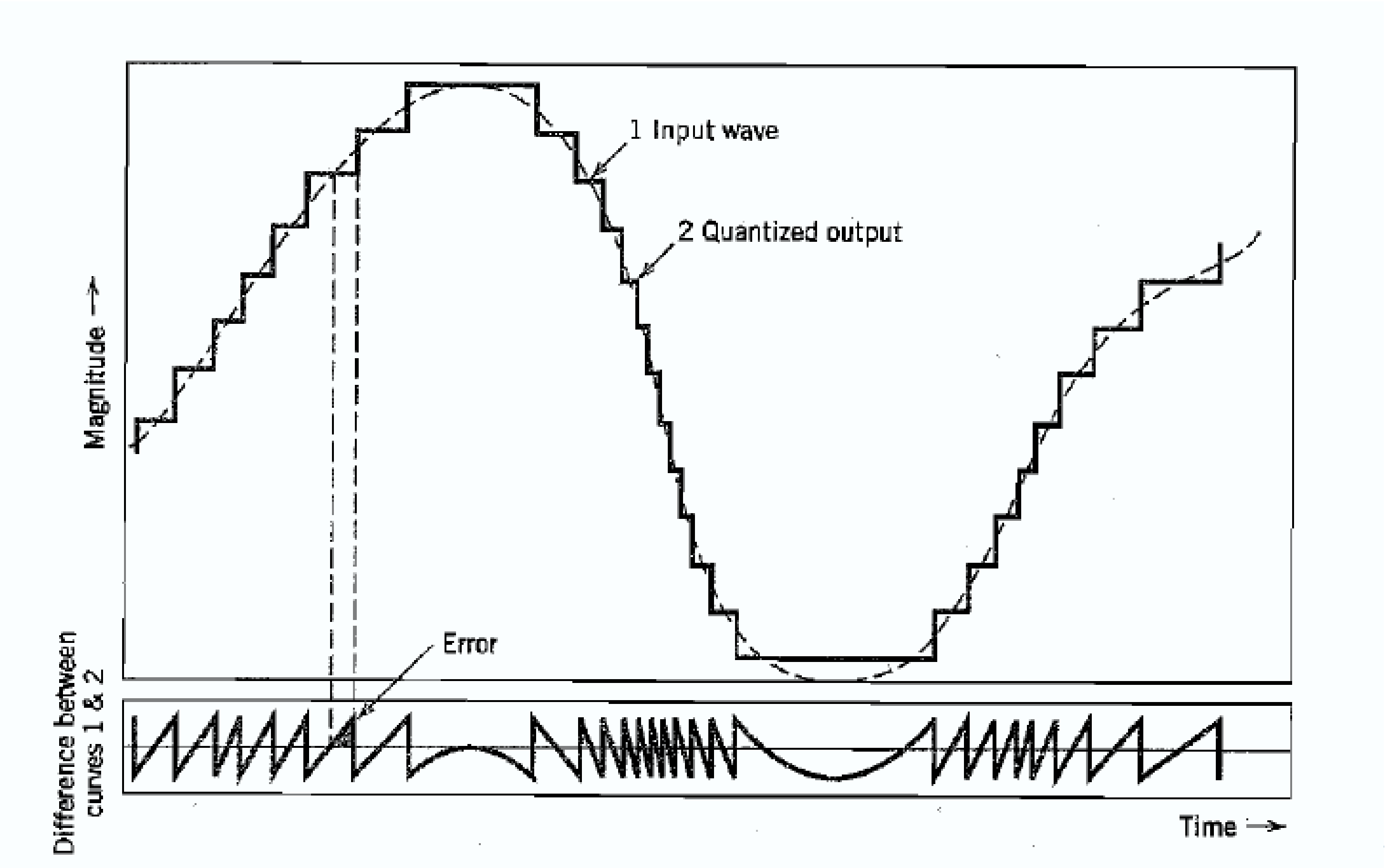
- Quantization regions: $\mathcal{R}_k,\ k = 1, \ldots, N$
- Quantization level for each $\mathcal{R}_k$: x_k
- Quantization:

$$\boxed{Q(x) = x_k, \text{ for all } x \in \mathcal{R}_k, k = 1, \ldots, N}$$



Quantization Process

The quantization introduces an error (difference from the input signal and the output signal), which is called **quantization noise**



Quantization Process

Average (mean square error) distorsion:

$$D = E\left[e^2\right] = E\left[(x - Q(x))^2\right]$$

$$SQNR = \frac{E\left[X^2\right]}{E\left[(x - Q(x))^2\right]}$$

(when x is a random variable, sample of a random process)

Quantization Process

- Range of the input samples = [-a, a]
- Number of quantization levels = $N = 2^{\nu}$
- Length of each quantization region $\Delta = \frac{2a}{N} = \frac{a}{2^{\nu-1}}$
- Quantized value are the midpoints of quantization regions.
- Quantization distortion (assume quantization error uniformly distributed on $(-\frac{\Delta}{2}, \frac{\Delta}{2})$)

$$E[e^2] = \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta} x^2 dx = \frac{\Delta^2}{12} = \frac{a^2}{2N^2} = \frac{a^2}{3 \cdot 4^{\nu}}$$

$$\boxed{SQNR = \frac{P_X}{E[e^2]} = \frac{3 \cdot 4^{\nu} P_X}{a^2} \overset{dB}{=} 10 \log_{10} \frac{P_X}{a^2} + 6\nu + 4.8}$$

One extra bit increases the SQNR by 6 dB!

Quantization Process

Homework:

Find the SQNR for a signal uniformly distributed on [-1,1] and quantized by a uniform quantizer with 256 levels.

Quantization Process

Quantizer saturation

If the signal in input to quantizer exceeds the operating range of it, the different between the input and the output signal becomes large and the quantized is operating in *saturation*.

Saturation errors might be large.

Saturation is avoided by the use of automatic gain control (AGC)
Which extends the operating range of the converter.

DIGITAL COMUNICATION SYSTEM

Digital Pulse Modulation

Encoding Process

The binary encoding process is to assign v bits to quantization levels $N = 2^v$
Since there are v bits for each sample and fs samples/second, we have a **bit rate** of

$$R = vf_s$$

Different type of binary encoding

Natural binary coding
Assign the values of 0 to N-1 to different quantization levels in order of increasing level value.

Gray coding
Adjacent levels differ only in one bit

Encoding Process

Level no	NBC	Gray code	Amplitude level
7	111	100	3.5
6	110	101	2.5
5	101	111	1.5
4	100	110	0.5
3	011	010	-0.5
2	010	011	-1.5
1	001	001	-2.5
0	000	000	-3.5

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
Pulse Code Modulation (PCM)

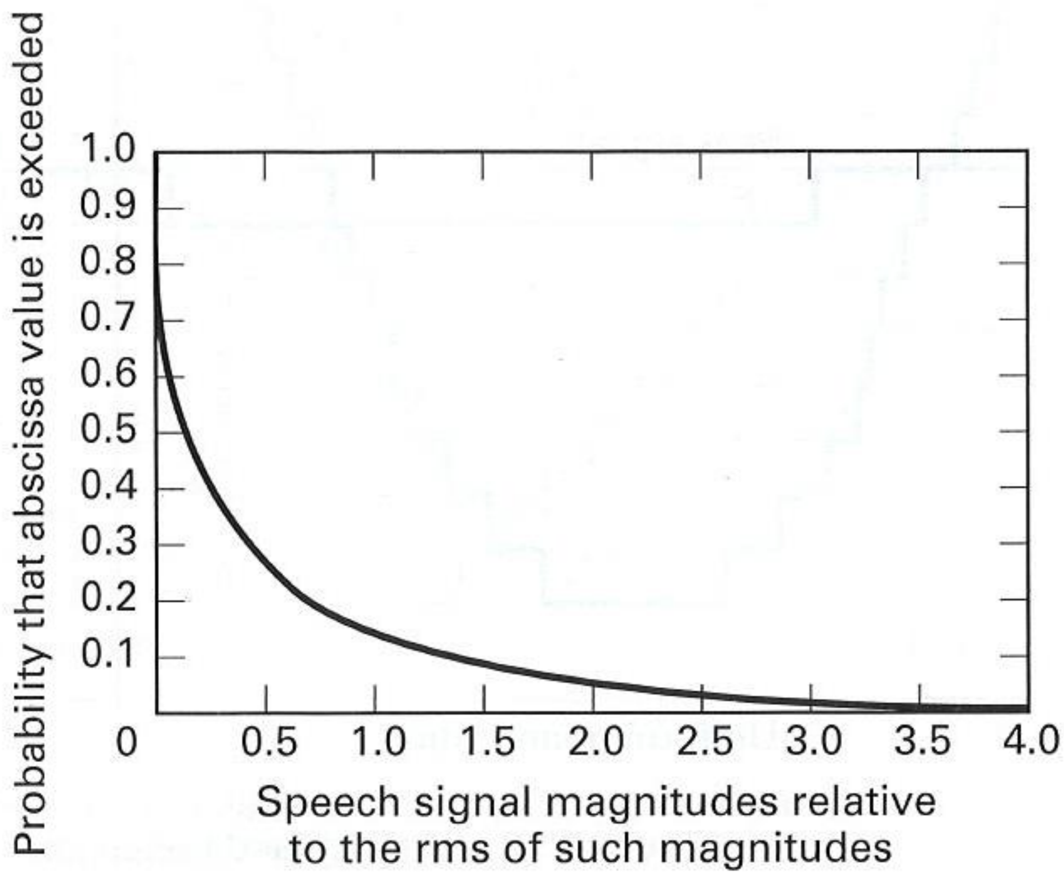
- Most widely used method for A/D conversion of audio source
- Block diagram of a PCM system

$$x(t) \longrightarrow \boxed{\text{Sampler}} \xrightarrow{\{x_n\}} \boxed{\text{Quantizer}} \xrightarrow{\{\hat{x}_n\}} \boxed{\text{Encoder}} \longrightarrow \{\ldots 0110 \ldots\}$$

- Uniform PCM: the quantizer is a uniform quantizer
- Bit rate
  - If a signal has a bandwidth of W and is sampled at f_s and v bits are used for each sampled signal, then $R_b = f_s v$ bits/Hz
- Bandwidth requirement:
  - With binary transmission (to be discussed in later chapters)

$$BW_{req} = \frac{R_b}{2} = \frac{f_s v}{2} \geq vW \text{ Hz}$$

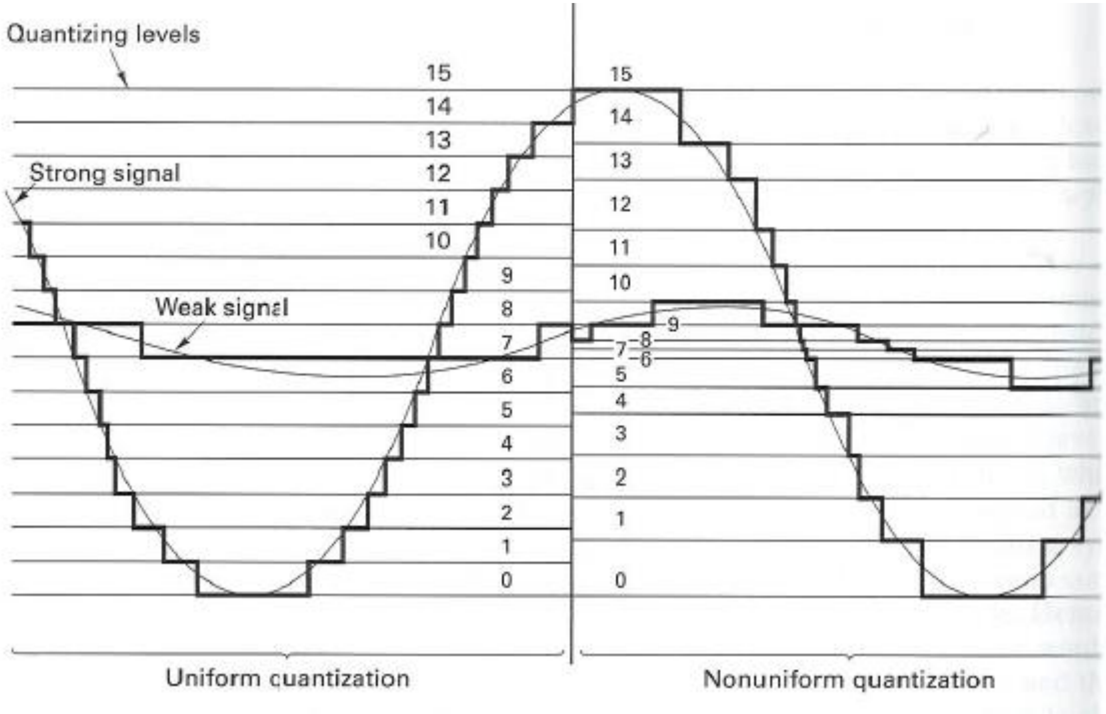
DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM



Very low speech volume predominate: 50% of the time, the voltage characterizing detected speech energy is less than ¼ of the rms of the amplitude. Large amplitude are very rare (15% of time).

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM

For speech waveform, there exists a higher probability for smaller amplitudes and a lower probability for larger amplitudes.



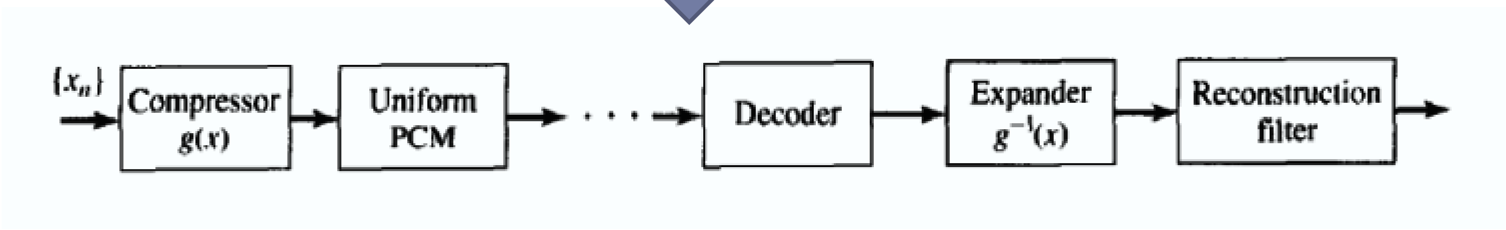
It makes sense to design a quantizer with more quantization regions at lower amplitudes and fewer quantization regions at large amplitudes.

Not uniform quantization

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM

The usual method to implement a not uniform quantizer is through the

COMPANDING (COMPressing-exPANDING)



First the signal is passed through a non linear filter that compress the large amplitude, then a uniform quantization is performed

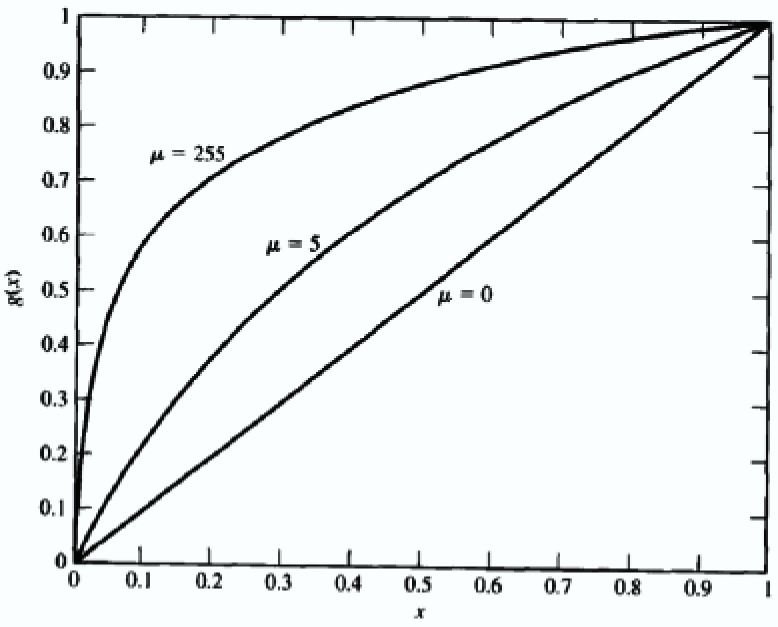
At the receiver, the inverse of this not linear operation is performed

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM

COMPANDING

- μ law compander

$$g(x) = \frac{\log(1+\mu|x|)}{\log(1+\mu)}\operatorname{sgn}(x), \quad |x| \le 1$$

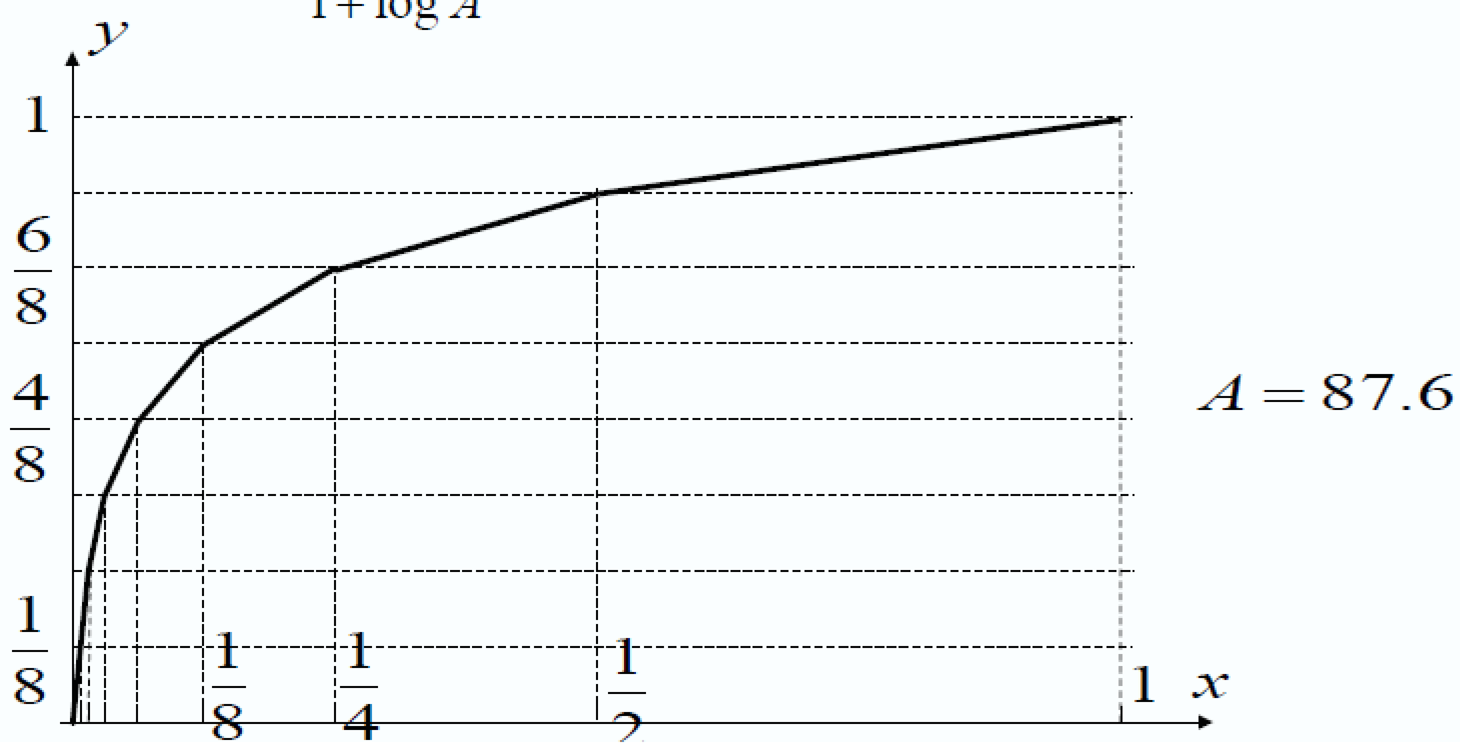


- The standard PCM system in US & Canada employs a compressor with $\mu = 255$ followed by a uniform quantizer with 8 bits/sample

COMPANDING

- A law compander

$$g(x) = \frac{1 + \log A|x|}{1 + \log A}\operatorname{sgn}(x), \quad |x| \le 1$$



Differential PCM (DPCM)

- For a bandlimited random process, the sampled values are usually correlated random variables

- This correlation can be exploited to improve the performance

- Differential PCM: quantize the difference between two adjacent samples.

- As the difference has small variation, to achieve a certain level of performance, fewer bits are required

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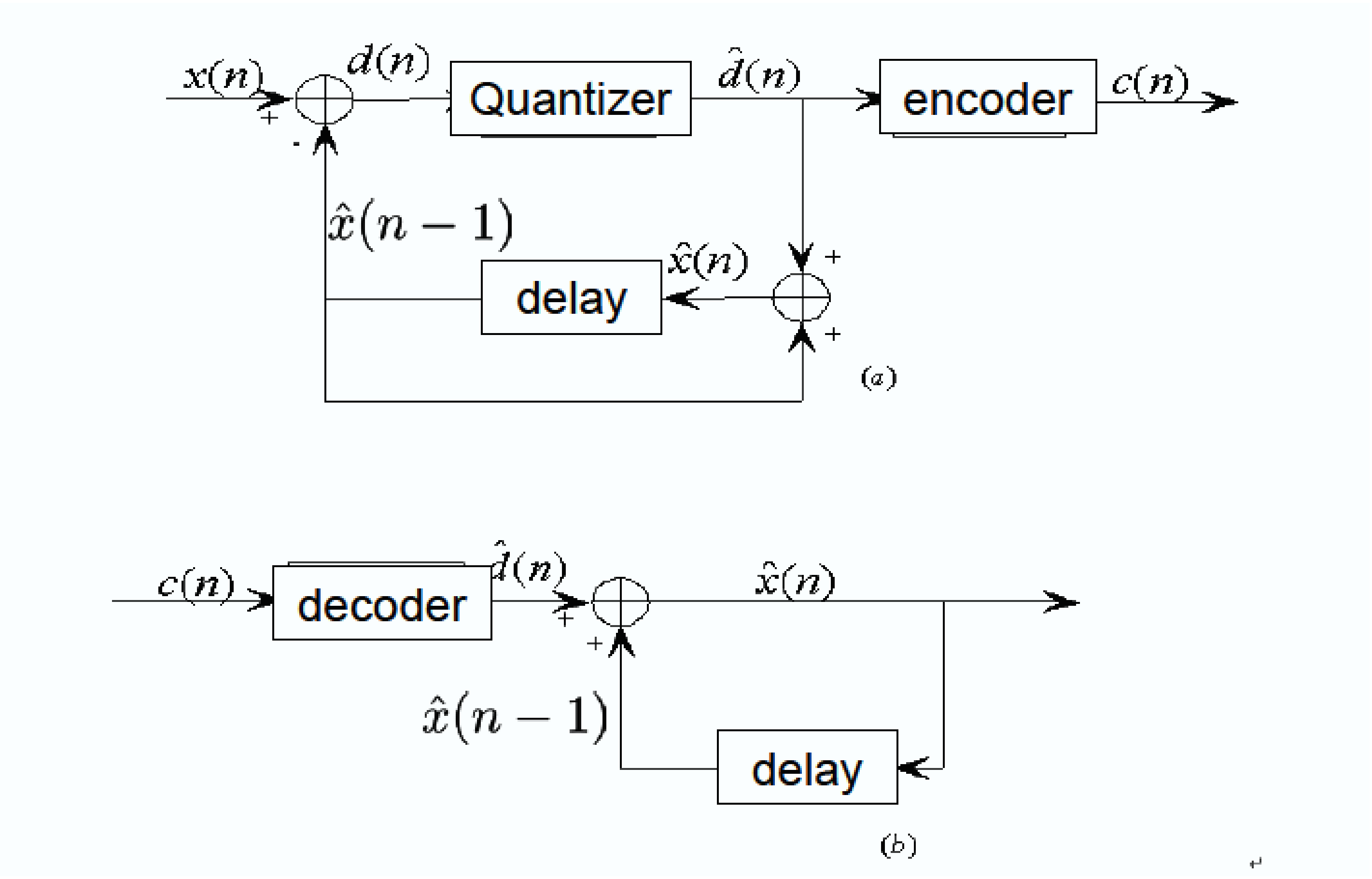
Digital Pulse Modulation
PCM

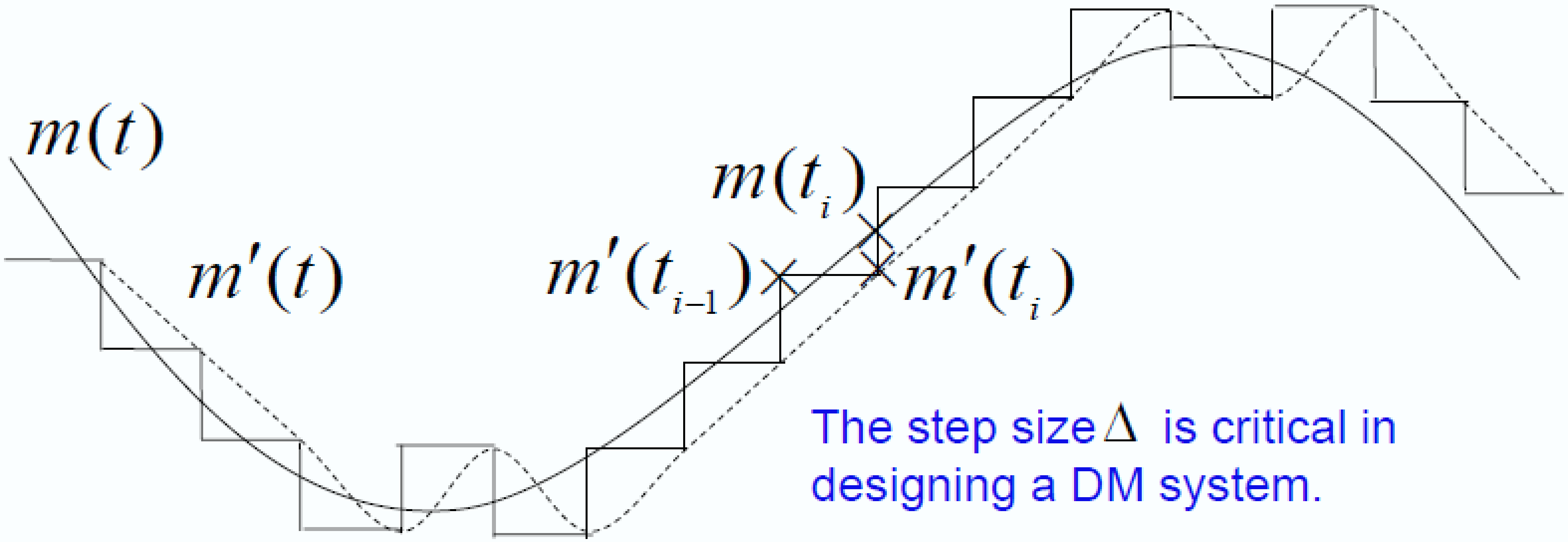
Differential PCM (DPCM)



DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM

Delta Modulation (DM)

- DM is a simplified version of DPCM having a two-level quantizer with magnitude $\pm\Delta$
- In DM, only 1-bit per symbol is employed. So adjacent samples must have high correlation.



The step size Δ is critical in designing a DM system.

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Digital Pulse Modulation
PCM

Delta Modulation (DM)

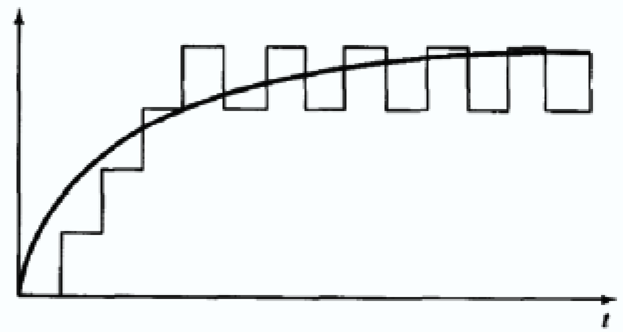


Figure 7.14 Large Δ and granular noise.

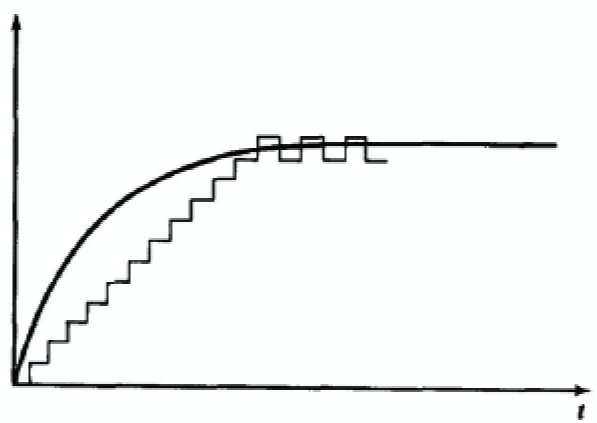
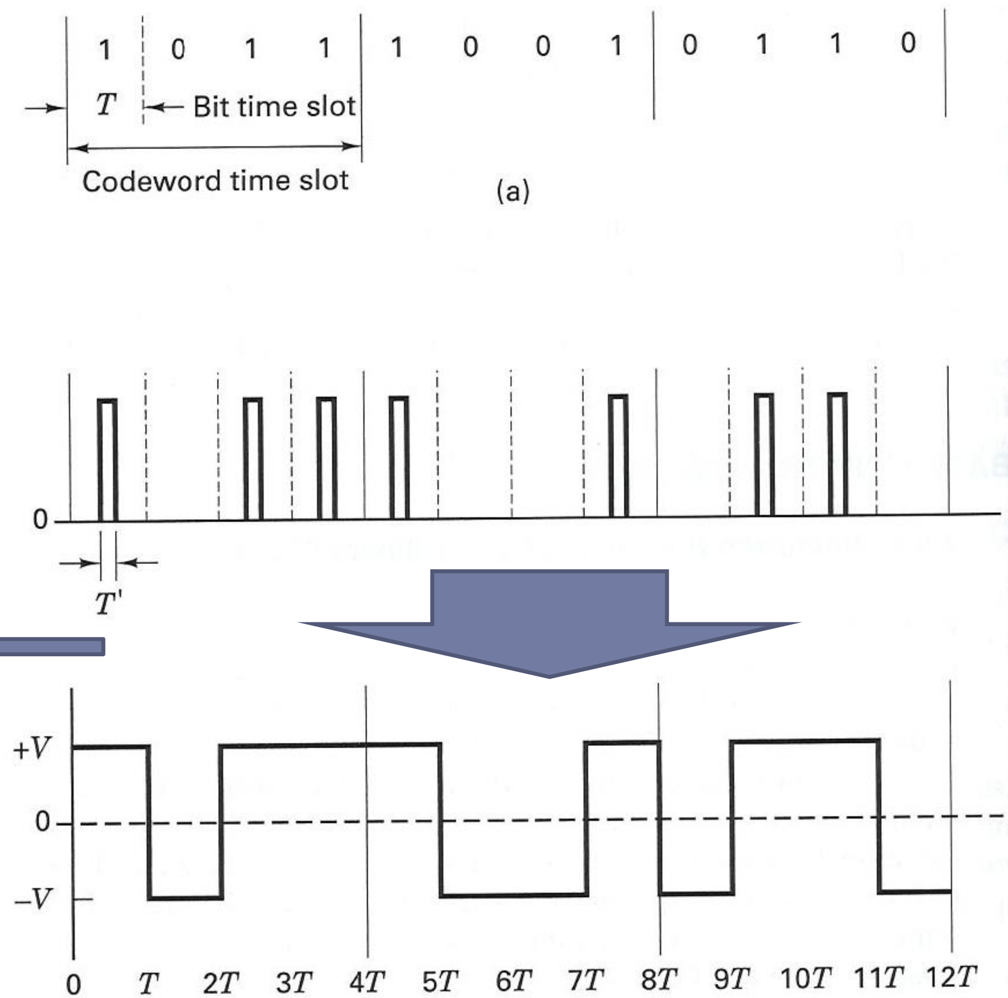


Figure 7.15 Small Δ and *slope overload* distortion.

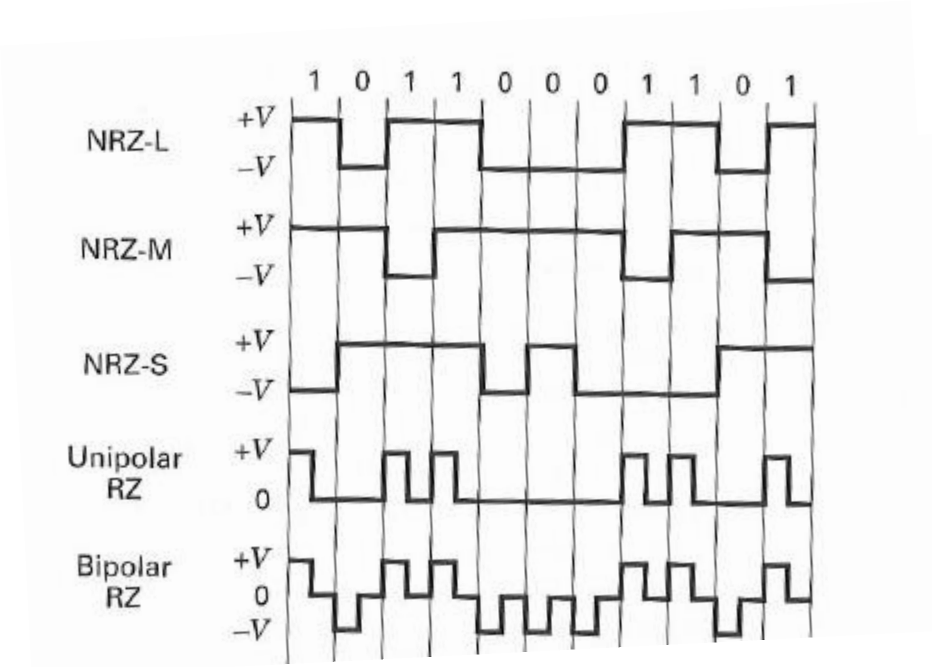
DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM waveforms

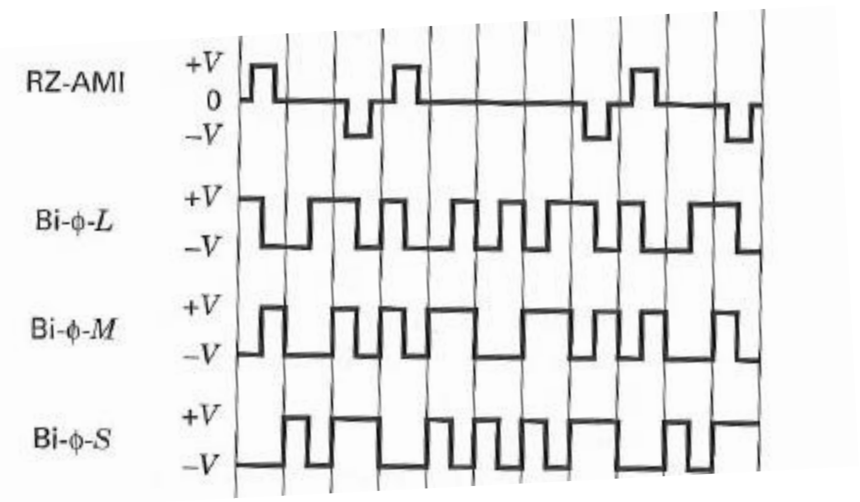


Different ways to implement this step:

- NRZ (Not-Return-to-Zero)
- RZ (Return-to-zero)
- Phase encoded
- Multilevel binary

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM waveforms





WHY SO MANY WAYS?

DIGITAL COMUNICATION SYSTEM
Digital Pulse Modulation
PCM waveforms

DC component: eliminating the dc component from the signal power spectrum is useful sometimes.
For instance, magnetic recording systems or systems using transforming coupling, have little sensitivity to very low frequency signal components ⟹ low frequency info could be lost
Self clocking: symbol and bit synchronization is required for any DCS. Manchester coding has a transition in the middle of every bit interval whether a one or a zero is being sent. This guaranteed transition provide the signal with an inherent clocking signal.
Bandwidth compression: each coding scheme is characterized by a different bandiwdth requirements.
Noise immunity: some of the waveforms are more immune to noise than others.

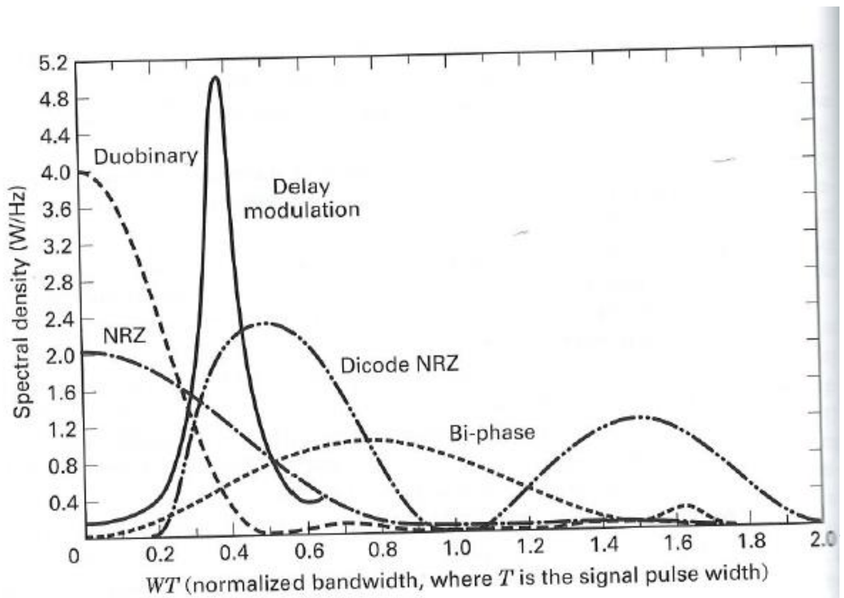


Figure 2.23 Spectral densities of various PCM waveforms.