Digital Communications
Transmission over bandlimited channels

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DIGITAL COMMUNICATION SYSTEM
Topics to be covered

- Bandlimited channels and Intersymbol Interference
- Signal design for bandilimited channels

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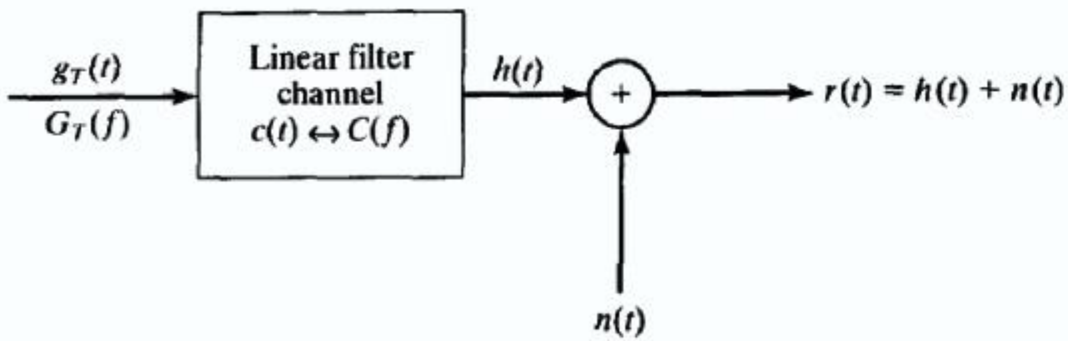
Transmission over a bandlimited channel

20MHz

5MHz, 10MHz
15MHz, 20MHz

- Modeled as a linear filter with frequency response limited to certain frequency range



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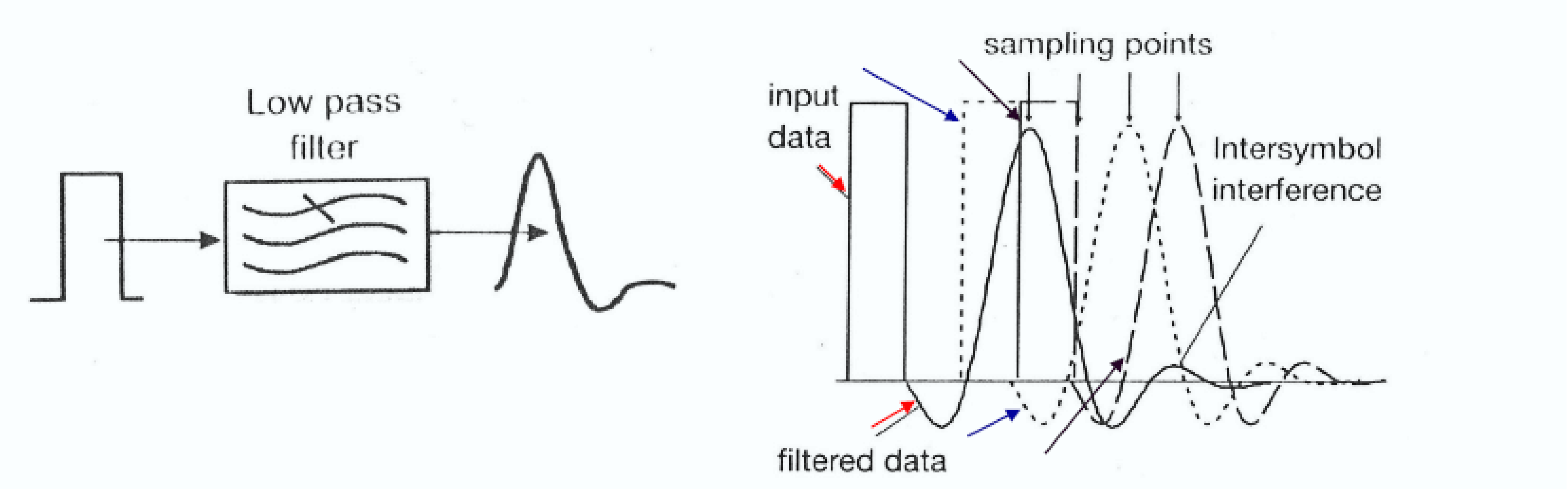
Transmission over a bandlimited channel



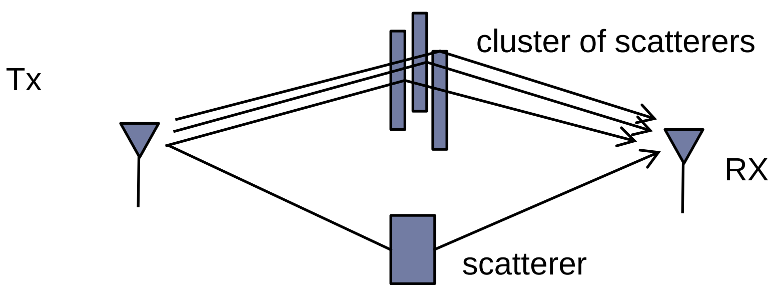
❑The filtering effect of the bandlimitedchannel will cause a **spreading** of individual data symbols passing through

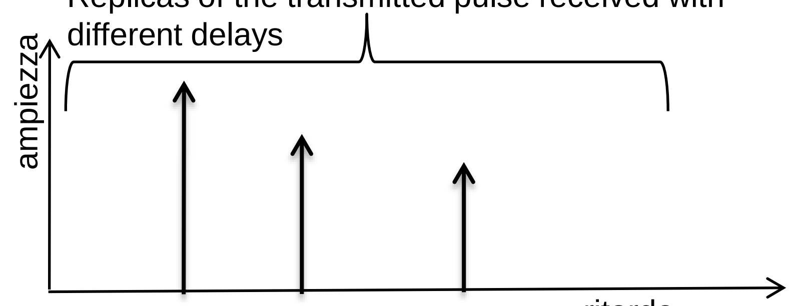
❑For consecutive symbols, this spreading causes part of the symbol energy to overlap with neighbouring symbols, causing **intersymbol interference (ISI)**

Intersymbol Interference

Example: wireless channel
Multipath phenomena

Due to multipath, if one pulse is transmitted, the received signal is a train of pulses which corresponds to the direct componenet (if present) and the replicas of the pulse reflected by single scatterers or by cluster of scatterers.





Note: cluster of scatterers are reflectors that produce replicas of the signal with delays that are similar and hence, they are not resolvible.

The two multipath components with $\tau 1$ and $\tau 2$ are resolvable if the difference between the delays is significantly higher than the inverse of the RX bandwidth.
Not resolvable components sum incoerently and give rise to multipath fading.

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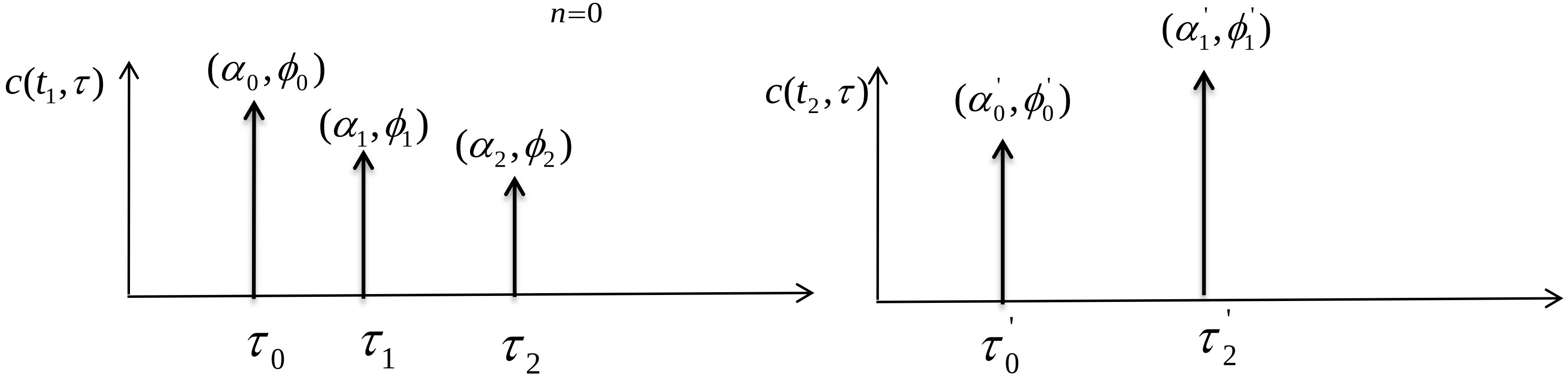
Baseband Pulse Transmission

Intersymbol Interference

Example: wireless channel
Multipath phenomena

Time variant impulse response of a multipath wireless channel

$$c(t,\tau)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\phi_n(t)}\delta(\tau-\tau_n(t))$$



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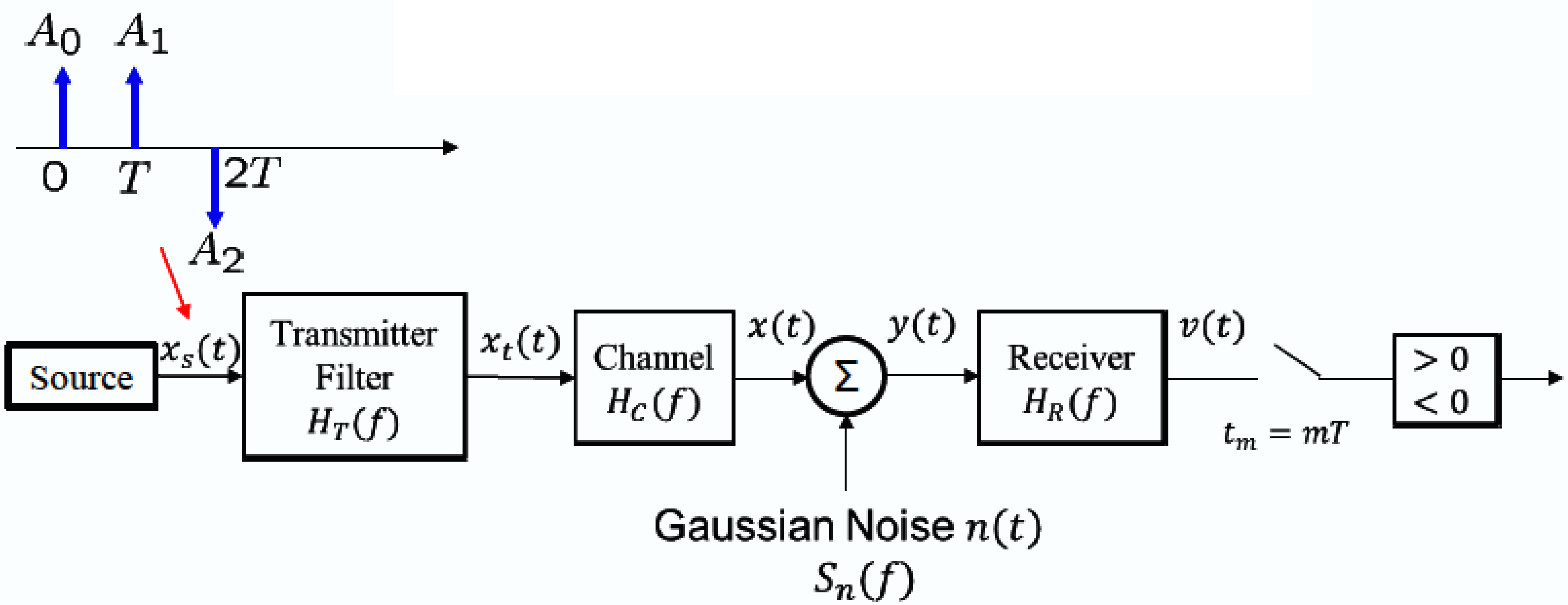

Input to tx filter
$$x_s(t) = \sum_{i=-\infty}^{\infty} A_i \delta(t - iT)$$

Output of tx filter
$$x_t(t) = \sum_{i=-\infty}^{\infty} A_i h_T(t - iT)$$

Output of rx filter
$$v(t) = x_s(t) * h_T(t) * h_c(t) * h_R(t) + n(t) * h_R(t)$$

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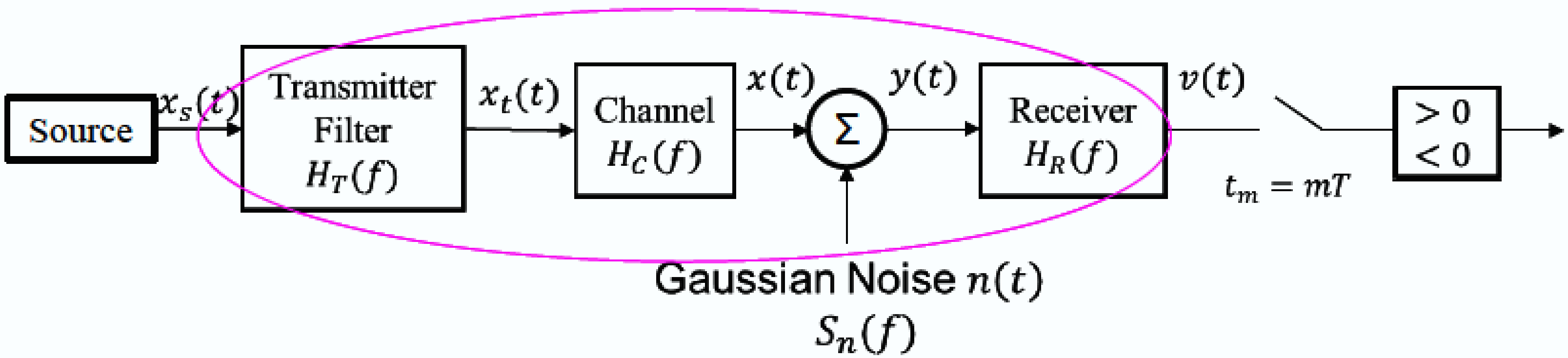
Transmission over a bandlimited channel



- Pulse shape at the receiver filter output

$$p(t) = h_T(t) * h_c(t) * h_R(t)$$

- Overall frequency response

$$P(f) = H_T(f)H_C(f)H_R(f)$$

- Receiving filter output

$$v(t) = \sum_{k=-\infty}^{\infty} A_k p(t - kT) + n_o(t)$$

$$n_o(t) = n(t) * h_R(t)$$

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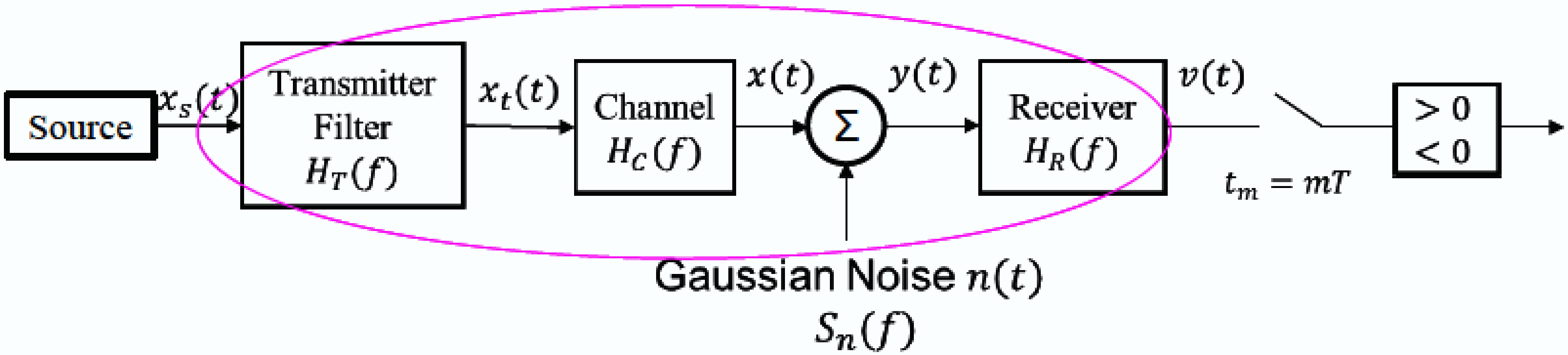
ISI



- Sample the rx filter output $v(t)$at $t_m = mT$ (to detect A_m)

$$v(t_m) = \sum_{k=-\infty}^{\infty} A_k p(mT - kT) + n_o(t_m)$$

$$= \underbrace{A_m p(0)}_{\text{Desired signal}} + \underbrace{\sum_{k \neq m}^{\infty} A_k p[(m-k)T]}_{\text{intersymbol interference (ISI)}} + \underbrace{n_o(t_m)}_{\text{Gaussian noise}}$$

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ISI
EYE DIAGRAM

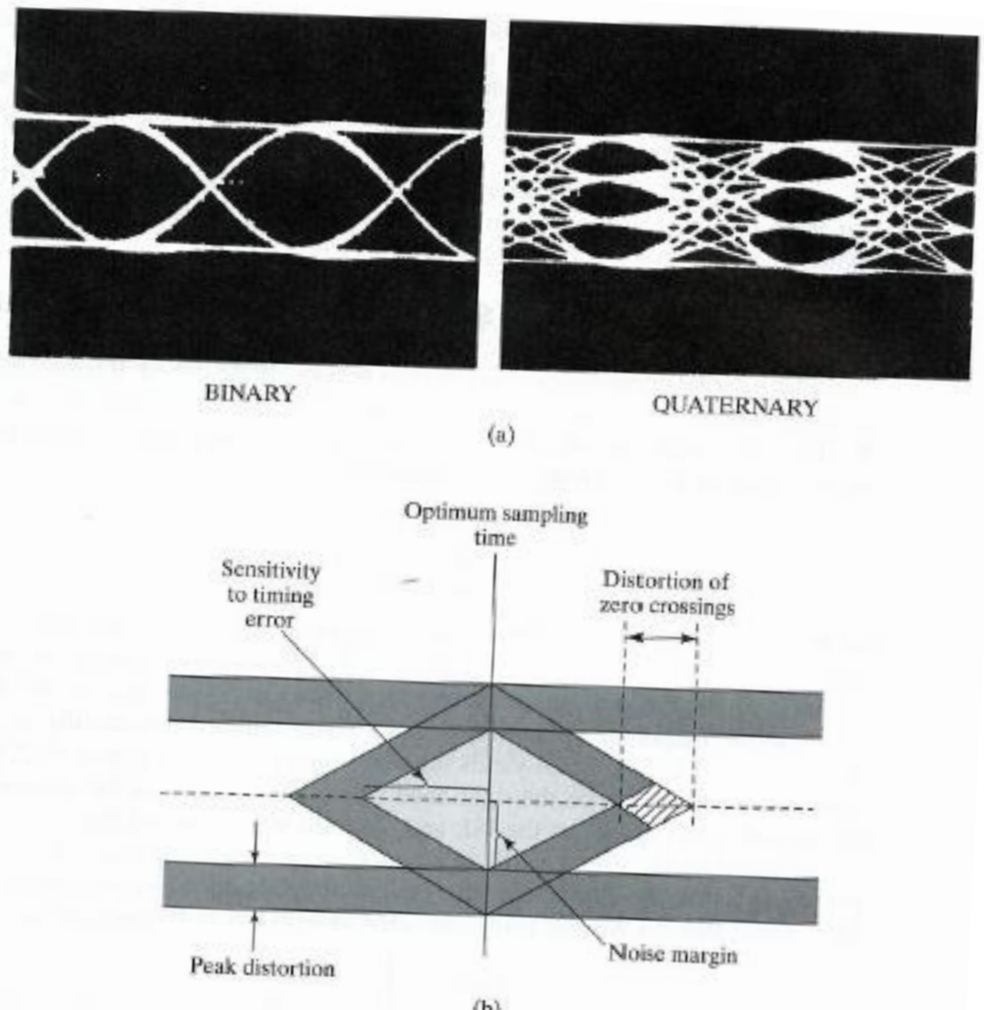

Figure 9.5 Eye patterns. (a) Examples of eye patterns for binary and quaternary PAM and (b) the effect of ISI on eye opening.

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ISI Minimization

Nyquist problem
(condition for a digital transmission without ISI)

Let us assume that there is no noise

$$v_m = v(mT) = \sum_i A_i p(mT - iT) + \cancel{n(mT)}$$

$$p(t) = h_T(t) * h_c(t) * h_R(t)$$

$$v_m = A_m p(0) + \underbrace{\sum_{i \neq m} A_i p((m-i)T)}_{\text{ISI}}$$

To have $v_m = A_m$

$$p(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Echos made to be zero at sampling points

Nyquist Condition for Zero ISI

A necessary and sufficient condition for p(t) to satisfy:

$$p(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Is that the Fourier Transform of p(t) must satisfy:

$$\sum_{k=-\infty}^{\infty} P\left(f + \frac{k}{T}\right) = \text{constant}$$

Proof in the following

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Baseband Pulse Transmission

Nyquist Condition for Zero ISI

$$p(t) = \int P(f)e^{j2\pi ft}df \quad \Rightarrow \quad p(nT) = \int P(f)e^{j2\pi fnT}df$$

$$p(nT) = \sum_{m=-\infty}^{\infty} \int_{(2m-1)/2T}^{(2m+1)/2T} P(f)e^{j2\pi nT}df =$$

$$= \sum_{m=-\infty}^{\infty} \int_{-1/2T}^{1/2T} P(v+\frac{m}{T})e^{j2\pi nT}df$$

Change of variable f = v+m/T

$$= \int_{-1/2T}^{1/2T} \left[\sum_{m=-\infty}^{\infty} P(v+\frac{m}{T}) \right] e^{j2\pi nT}dv$$

$$= \int_{-1/2T}^{1/2T} Z(v)e^{j2\pi nT}dv$$

Nyquist Condition for Zero ISI

$$Z(f) = \sum_{m=-\infty}^{\infty} P(f + \frac{m}{T})$$

Which is periodic with period 1/T so it can be expanded in terms of its Fourier series coefficients as

$$Z(f) = \sum_{n=-\infty}^{\infty} z_n e^{j2\pi nfT}$$

$$z_n = T\int_{-1/2T}^{1/2T} Z(f)e^{-j2\pi nfT} df$$

$$z_n = Tp(-nT) \quad \Longrightarrow \quad z_n = \begin{cases} T & \text{n} = 0 \\ 0 & \text{n} \neq 0 \end{cases}$$

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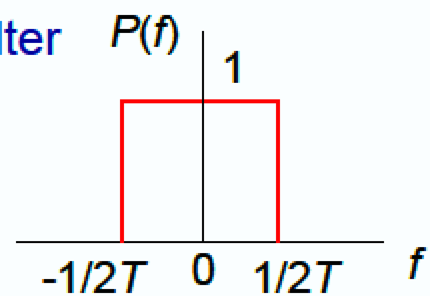
Baseband Pulse Transmission

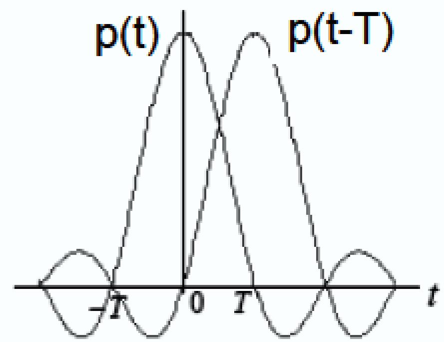
Nyquist problem: IDEAL SOLUTION

- Nyquist's first method for eliminating ISI is to use

$$P(f) = \begin{cases} 1 & |f| < \frac{1}{2T} \\ 0 & \text{otherwise} \end{cases} \Longleftrightarrow p(t) = \frac{\sin(\pi t / T)}{\pi t / T} = \text{sinc}\left(\frac{t}{T}\right)$$

"brick wall" filter





$$B_0 = \frac{1}{2T} = \frac{R_s}{2}$$ = Nyquist bandwdith,

The minimum transmission bandwidth for zero ISI. A channel with bandwidth B_0 can support a max. transmission rate of $2B_0$ symbols/sec

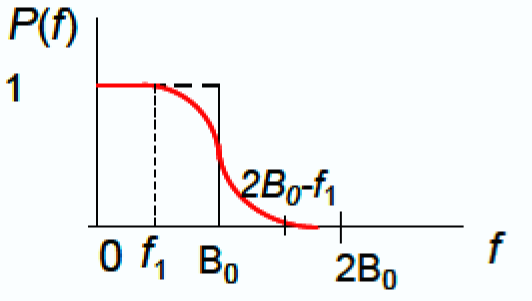
Nyquist problem: IDEAL SOLUTION

- Challenges of designing such $p(t)$ or $P(f)$
  - $P(f)$ is physically unrealizable due to the abrupt transitions at $\pm B_0$
  - $p(t)$ decays slowly for large t, resulting in little margin of error in sampling times in the receiver.
  - This demands accurate sample point timing - a major challenge in modem / data receiver design.
  - Inaccuracy in symbol timing is referred to as timing jitter.

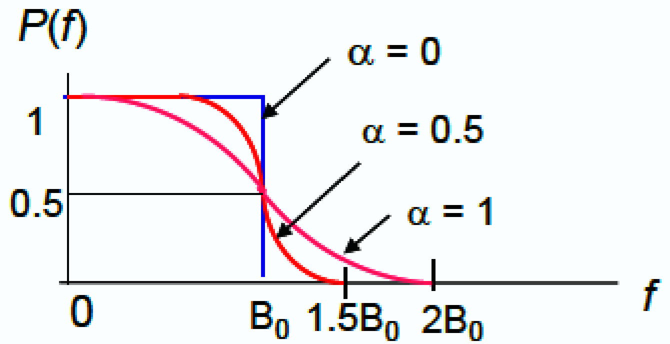
Nyquist problem: PRACTICAL SOLUTIONS RAISED COSINE SPECTRUM

- $P(f)$ is made up of 3 parts: passband, stopband, and transition band. The transition band is shaped like a cosine wave.

$$P(f)=\begin{cases} 1 & 0 \le |f| < f_1 \\ \frac{1}{2}\left\{1+\cos\left[\frac{\pi(|f|-f_1)}{2B_0-2f_1}\right]\right\} & f_1 \le |f| < 2B_0 - f_1 \\ 0 & |f| \ge 2B_0 - f_1 \end{cases}$$



Nyquist problem: PRACTICAL SOLUTIONS
RAISED COSINE SPECTRUM



Roll-off factor

$$\alpha = 1 - \frac{f_1}{B_0}$$

- The sharpness of the filter is controlled by α.
- Required bandwidth $B = B_0(1+\alpha)$

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Baseband Pulse Transmission

Nyquist problem: PRACTICAL SOLUTIONS
RAISED COSINE SPECTRUM

- Benefits of small a
  - Higher bandwidth efficiency
- Benefits of large a
  - simpler filter with fewer stages hence easier to implement
  - less sensitive to symbol timing accuracy