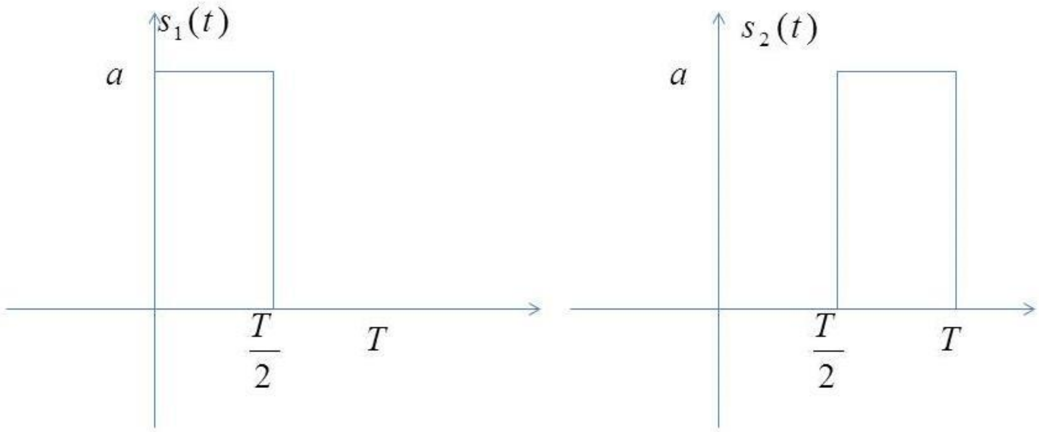
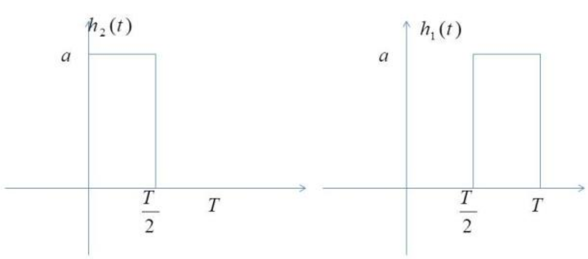
The two pulses shown in the figure below are employed in a binary orthogonal signaling scheme to be used in a digital communication system with AWGN channel with noise energy *N0/2*.



(a) Sketch carefully the optimum receiver (and the impulse response of the filters are included in the receiver).
(b) Assuming that a correlator receiver is employed, due to a timing error, the output is incorrectly sampled at $t = 0.75T$. What is the increase in the probability of a bit error, Pb, compared to that of an ideal system?

Solution

(a) Matched filters

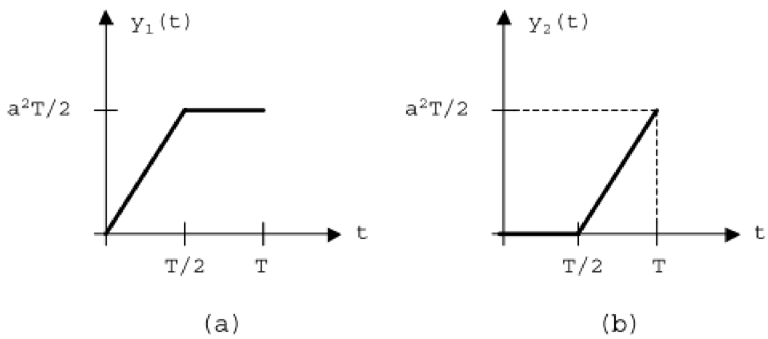


(b)

When $s_1(t)$ is sent, Y_1 is a r.v. with mean $m_1 = E_1 = \frac{a^2 T}{2}$ and variance $\sigma^2 = E_1 \frac{N_0}{2}$ and Y_2 is a r.v. with mean $m_2 = 0$ and variance $\sigma^2 = E_2 \frac{N_0}{2} = \frac{a^2 T}{2} \frac{N_0}{2}$and

(c)

Correlator outputs are:



(a) First output when $s_1(t)$ sent. (b) Second output when $s_2(t)$ sent.

When sampled at t=T, the mean and the variance are the one of the matched filter (see point (b)). When sampled at 0.75T, the mean of the first correlator does not change (always E1) but the mean of the second correlator is lower and equal to: (0.75T-T/2)a^2=(3/4-1/2)Ta^2=a^2T/4=E2/2 (half of the energy than in the previous case).

Also the variance of the noise is:

$$\sigma_1^2 = (0.5)\frac{N_0}{2}\left(\frac{a^2T}{2}\right) = \frac{N_0}{2}\left(\frac{a^2T}{4}\right)$$

From this observation it follows that

$$\begin{aligned} P[\text{error}|\bar{s}_1 \text{ sent}] &= Q\left(\sqrt{\frac{E}{N_0}}\right) = Q\left(\sqrt{\frac{a^2T}{2N_0}}\right), \text{ and} \\ P[\text{error}|\bar{s}_2 \text{ sent}] &= Q\left(\sqrt{\frac{E}{2\,N_0}}\right) = Q\left(\sqrt{\frac{a^2T}{4N_0}}\right). \end{aligned}$$

As a result, the average probability of a bit error becomes

$$P_b = \frac{1}{2}\left[Q\left(\sqrt{\frac{a^2T}{2N_0}}\right) + Q\left(\sqrt{\frac{a^2T}{4N_0}}\right)\right].$$

In conventional binary orthogonal signaling,

$$P_b = Q\left(\sqrt{\frac{a^2T}{2N_0}}\right).$$

The increase in the probability of a bit error, due to the timing error, is

$$\Delta_{P_b} = \frac{1}{2}\left[Q\left(\sqrt{\frac{a^2T}{4N_0}}\right) - Q\left(\sqrt{\frac{a^2T}{2N_0}}\right)\right].$$