Mobile Wireless Networks

Chapter 2: Multiple Antenna Techniques for cellular system

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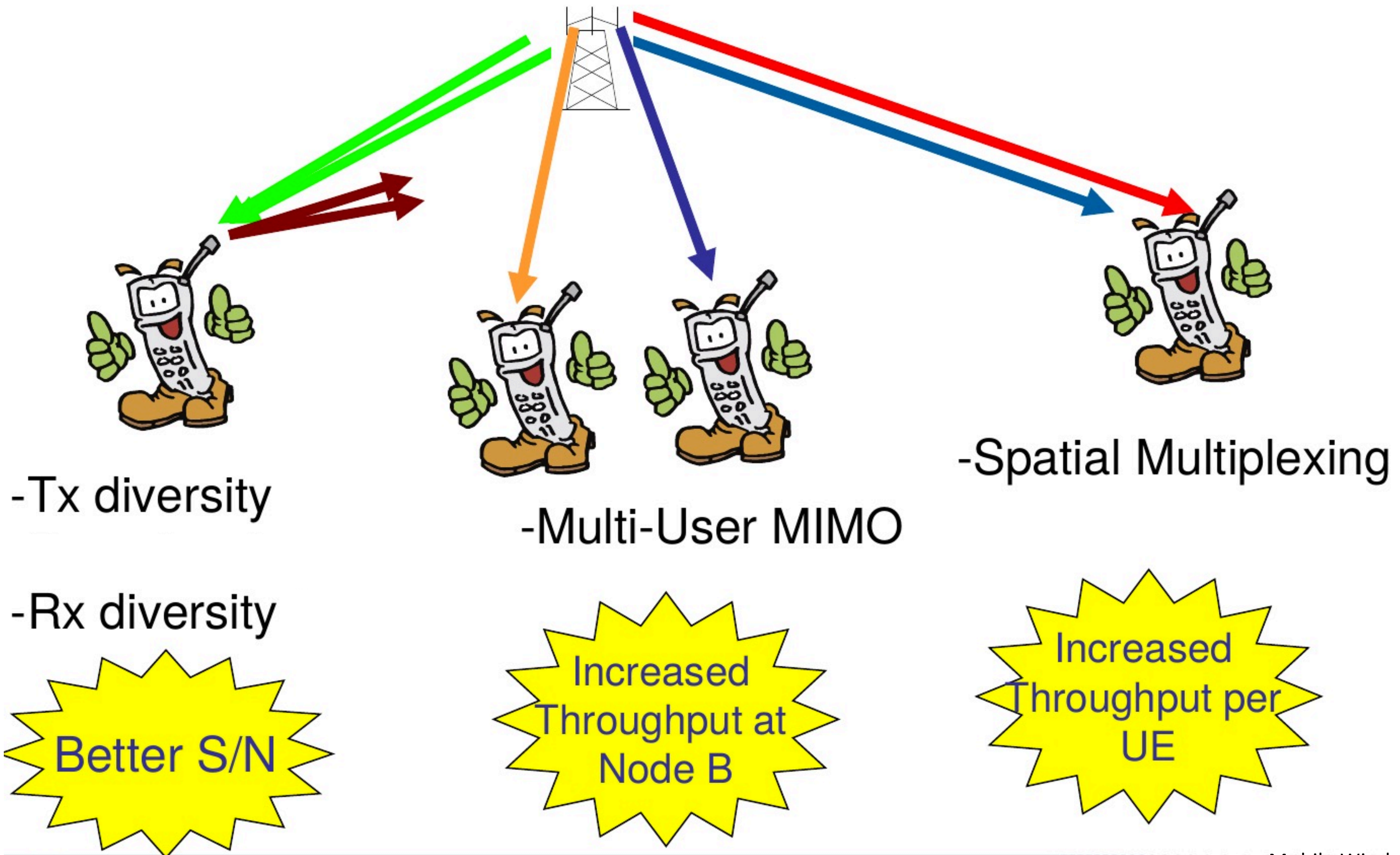
Sources:

1. *Christopher Cox, An Introduction to LTE: LTE, LTE-Advanced, SAE, VoLTE and 4G Mobile Communications: Second Edition, 2*

Rev 2.0

MULTIPLE ANTENNAS

Why


-Tx diversity
-Rx diversity
Better S/N
-Multi-User MIMO
Increased Throughput at Node B
-Spatial Multiplexing
Increased Throughput per UE

Objectives

- **TX/RX Diversity**
  - **Increases the received signal power** and reduces the amount of fading
  - Many TX/RX antennas for the same signal
  - Used since the early days of mobile communications, we will only review it briefly
- **Single User Spatial Multiplexing (aka Single user MIMO, SU-MIMO)**
  - **Improves single user bit-rate**
  - Many antennas for parallel data streams towards the *same* user
- **Beam forming (Multi-User Spatial Multiplexing or Multi User MIMO, MU-MIMO)**
  - **Improves cell-level bit-rate**
  - Multiple synthetic directional antennas for for parallel data streams towards *different* users

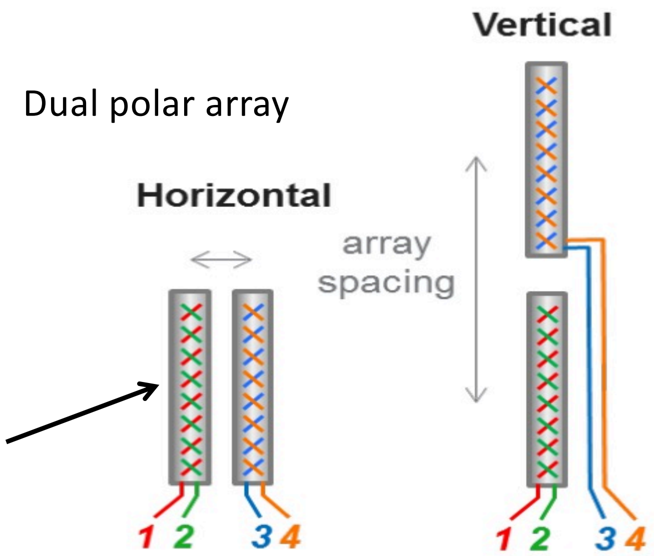
TX/RX DIVERSITY

Antenna Placement

- TX/RX Diversity and Spatial Multiplexing techniques leverage the not correlation among the couples of receiving and transmitting antennas
- Two contrasting requirements:
  - Place the transmitting (receiving) multiple antennas far enough each other to de-correlate the tx-rx paths
  - Place transmitting (receiving) antennas close enough to each other so that the RF coverage area is nearly identical
- Rule of thumb: antenna separation few wavelengths

Antenna Placement

Dual polar array

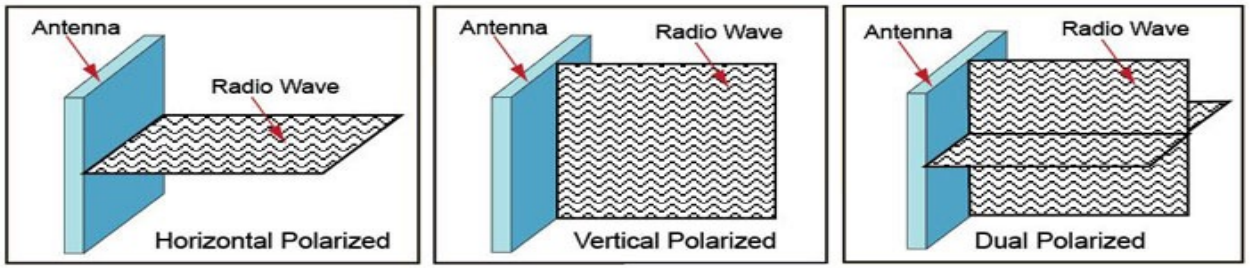


Cross polar element 1/2 and 3/4 are always de-correlated

X-pol Array Configuration	Array Spacing (Macro environment)	Antenna Port Correlation	
		p(1,2) p(3,4)	p(1,3) p(2,4)
H — narrow	< 1λ (43cm@700MHz)	Low	High
H — wide	> 4λ (1.7m@700MHz)	Low	Low
V — Narrow	< 8λ (3.4m@700MHz)	Low	High
V — Wide	> 12λ (5.1m@700MHz)	Low	Low

1:1 port -element mapping in this case

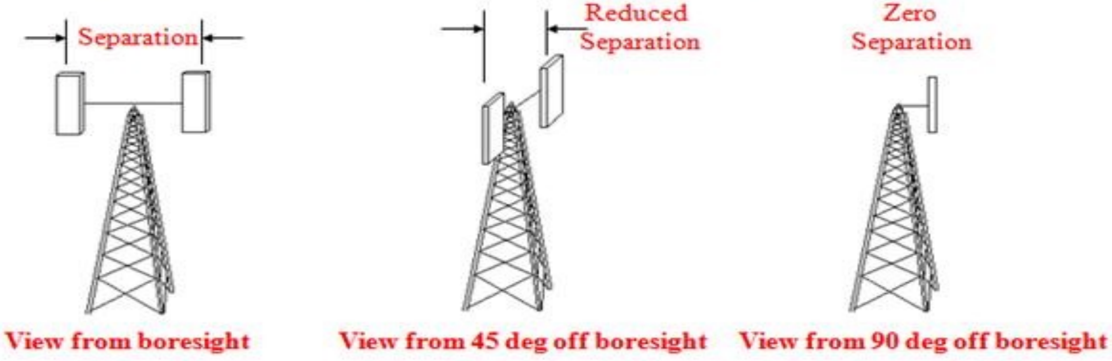
Antenna Elements

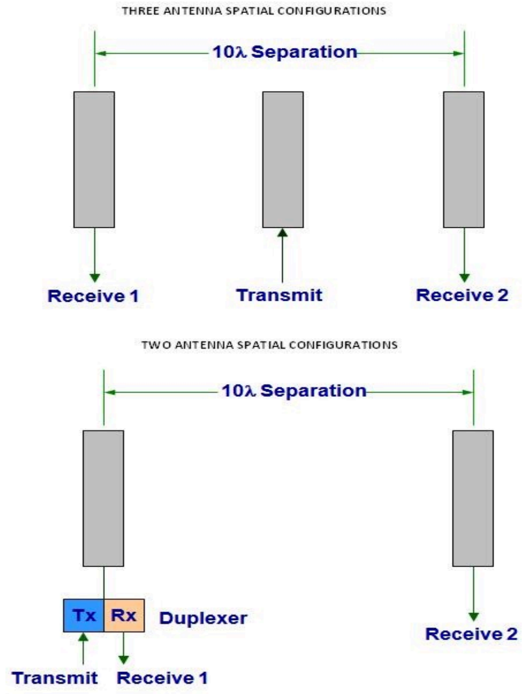


- A horizontally polarized antenna will not communicate with a vertically polarized antenna and vice versa

Antenna Placement example

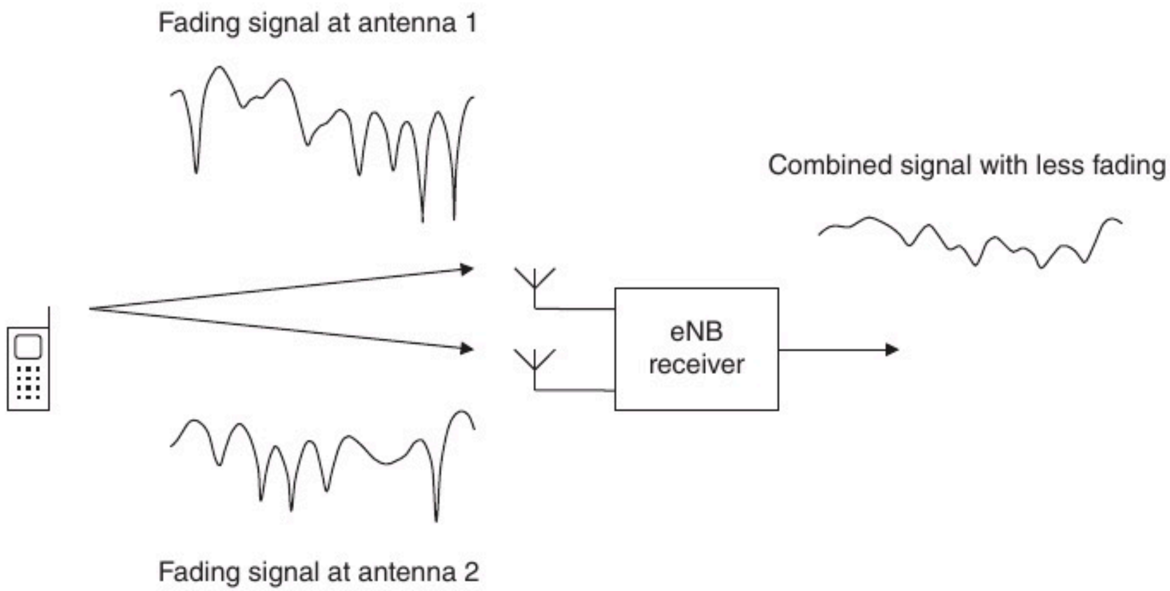
	Horizontal Separation			Vertical Separation		
	d/λ	900MHZ	1800MHZ	d/λ	900MHZ	1800MHZ
Separation	10	3.3m	1.7m	17	5.7m	2.8m



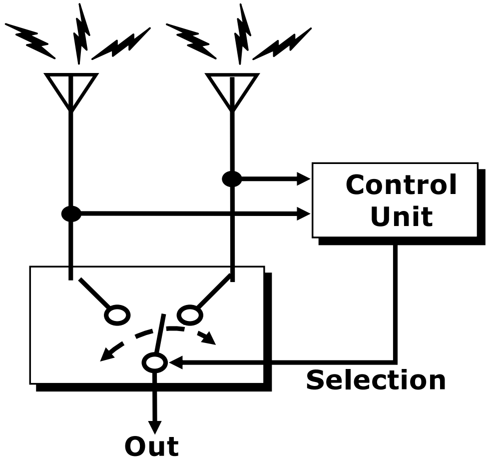


Receive Diversity: Concepts

- Use two or more receiving antennas and one transmitter
- If antennas are far enough apart, multipath impairments suffered by the two signals are rather different.
- Optima combination can reduces error rate.



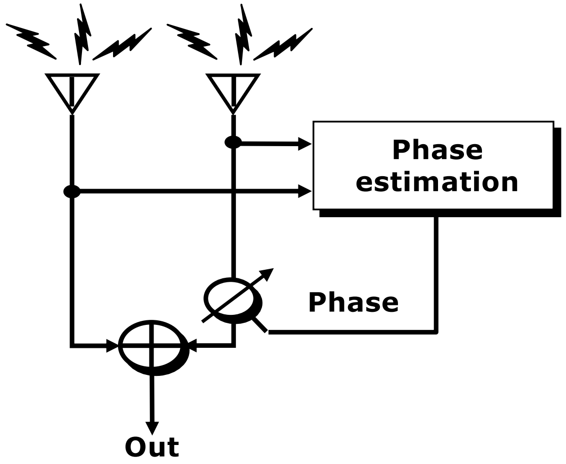
Receive Diversity: Combining



Selection Combining

$$Y_C = \begin{cases} Y_1 & ; Y_1 > Y_2 \\ Y_2 & ; Y_2 > Y_1 \end{cases}$$

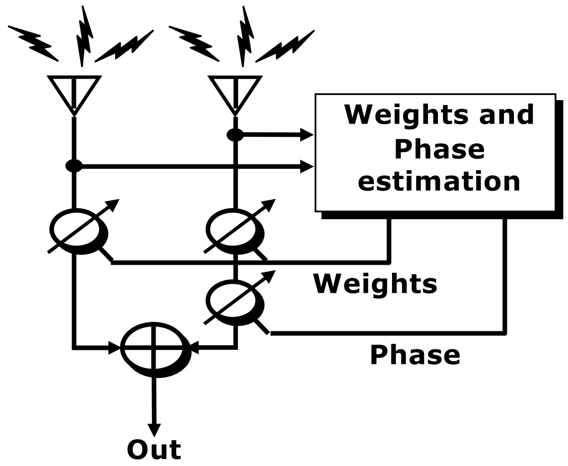
Choose the best



Equal Gain Combining

$$Y_C = \frac{Y_1 + Y_2}{2}$$

Simple average



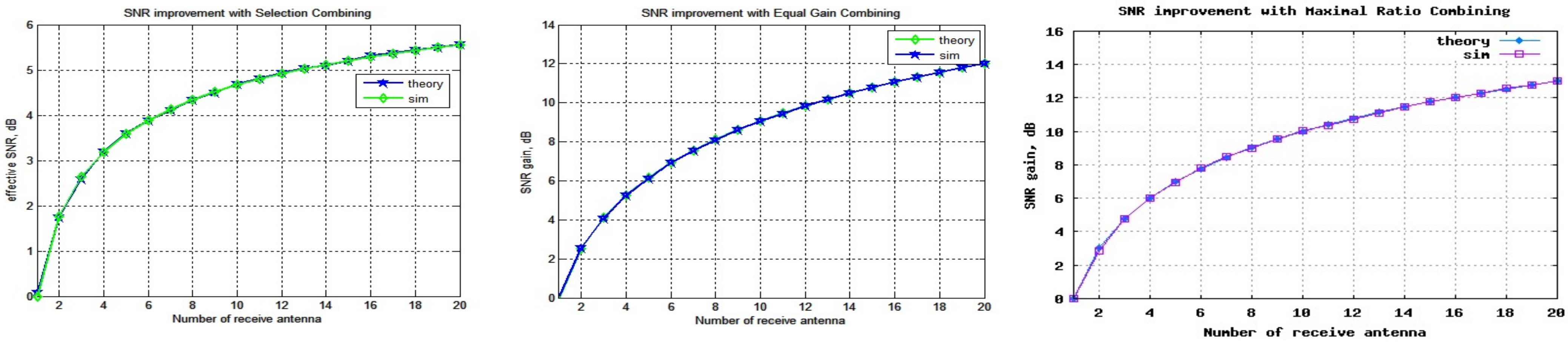
Maximal Ratio Combining

$$Y_C = a_1 Y_1 + a_2 Y_2$$

Weighted average

A sub-optimal version of selection combining is switch-and-stay combining in which alternate antenna are chosen if signal falls below a certain threshold

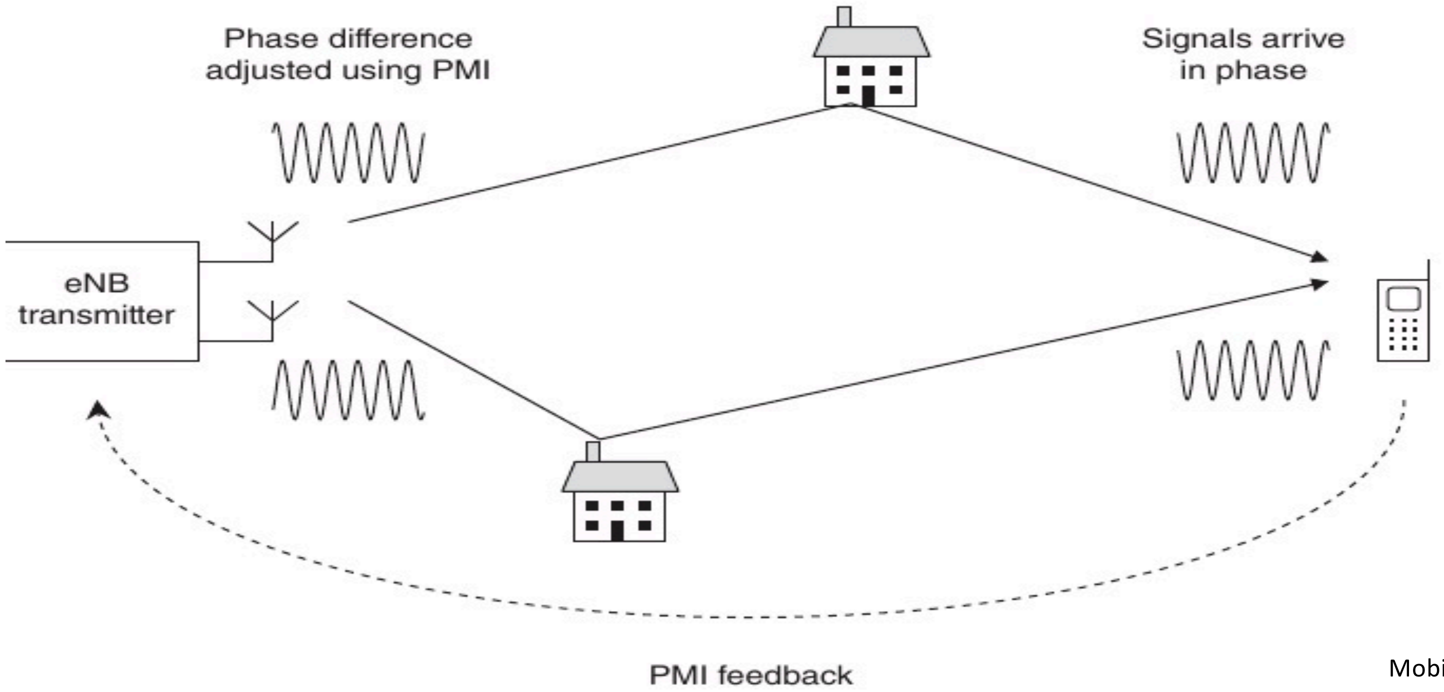
Diversity Gain



Single Transmitter, AWGN noise
(http://www.dsplog.com)

Transmit Diversity: Closed Loop

- Reduces the amount of fading by using two or more antennas at the transmitter.
- Similar to receive diversity, but with a crucial problem: the signals add together at the single receive antenna, which brings a risk of destructive interference.

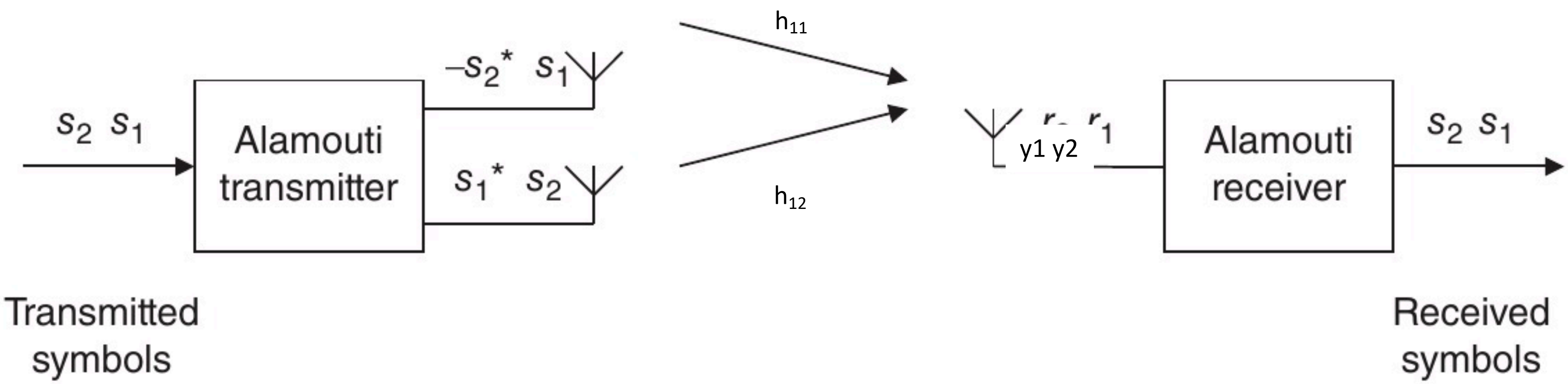


Transmit Diversity: Closed Loop

- The use of a “closed loop” is a way to solve the problem
- The transmitter sends the copies applying a phase shift to them so as combination at the receiving side is constructive (in-phase) rather than destructive (180 phase shift)
- The phase shift is communicated by the receiver through a feedback loop
  - In LTE the feedback is named Pre-Coding Matrix Indicator (PMI)
- The best choice of PMI also depends on the position of the mobile, so a fast moving mobile will have a PMI that frequently changes.
- Feedback loop introduces time delays into the system, so in the case of fast moving mobiles, the feedback may be out of date. For this reason, closed loop transmit diversity is only suitable for mobiles that are moving sufficiently slowly. For fast moving mobiles, it is better to use the open loop technique.

Transmit Diversity: Open Loop

- Alamouti technique. Does not require feedbacks
- Two antennas for sending two symbols s_1 and s_2 in two steps : s_1, s_2 and s_1^*, $-s_2^*$
- The symbol * indicates that the transmitter should change the sign of the quadrature component, in the process of complex conjugation.



Transmit Diversity: Alamouti math

- H matrix has determinant not null, thus can be inverted
- Receiver estimate $\hat{H}$ through reference symbols

Received signal at time t_1 , $y_1 = s_1 h_{11} + s_2 h_{12} + n_1$

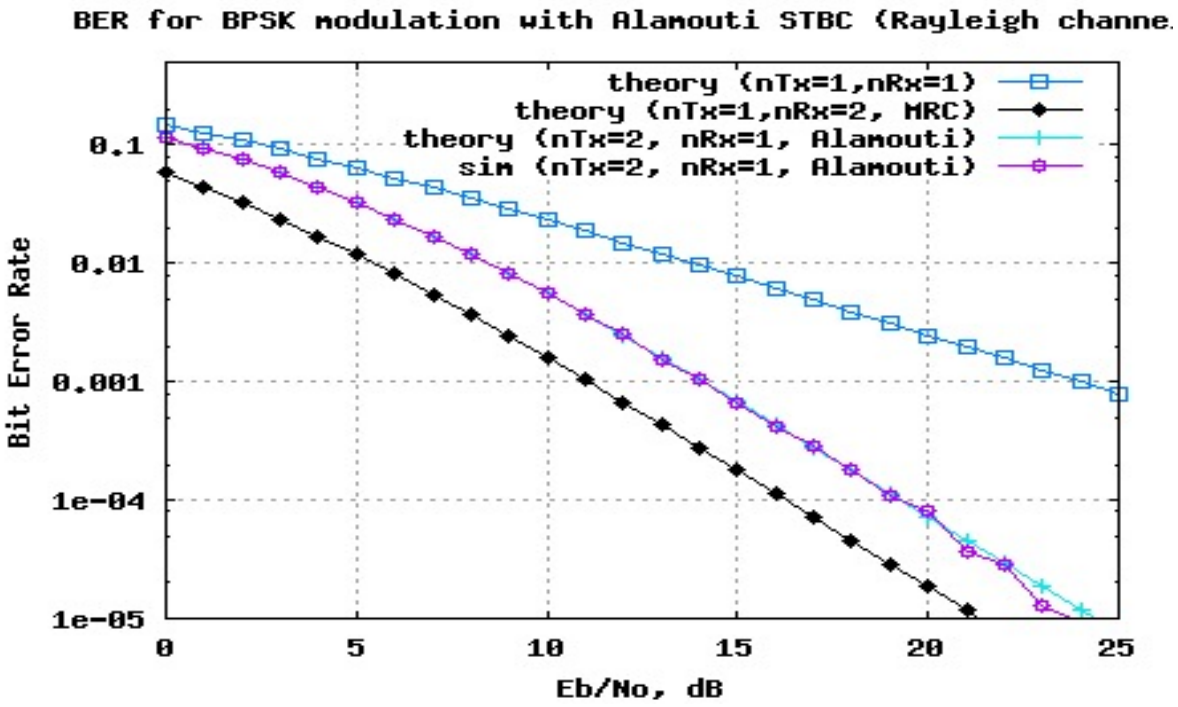
Received signal at time t_2 , $y_2 = -s_2^* h_{11} + s_1^* h_{12} + n_2$

Received signal vector
$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Decoded symbols
(assuming perfect channel estimation)
$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \hat{H}^{-1} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \hat{H}^{-1} H \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \hat{H}^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \hat{n}_1 \\ \hat{n}_2 \end{bmatrix}$$

Transmit Diversity: Alamouti performance



(http://www.dsplog.com)

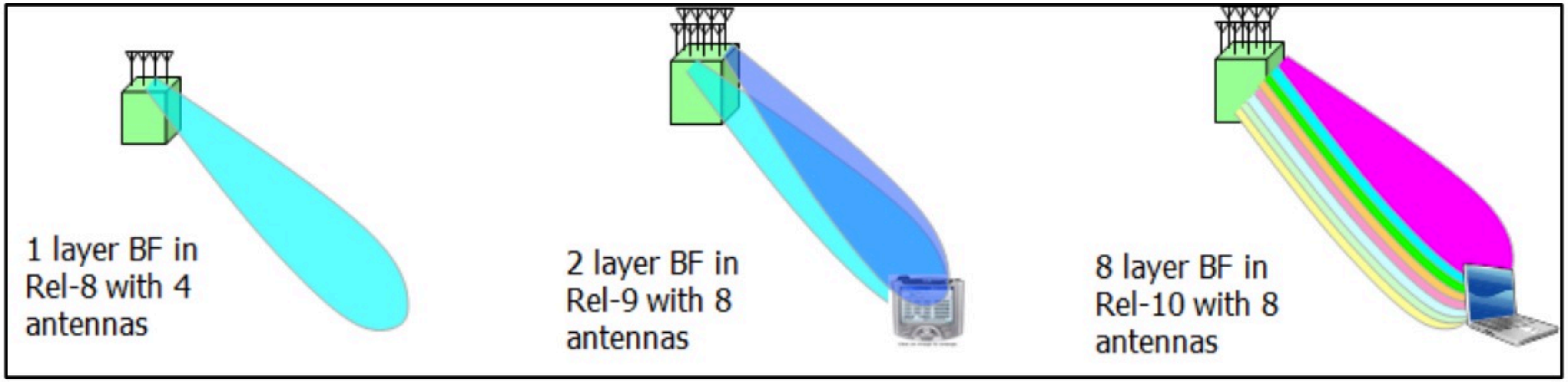
- Compared to the BER plot for nTx=1, nRx=2 Maximal Ratio Combining, we can see the Alamouti Space Time Block Coding has around 3dB poorer performance.
- Receiver diversity more powerful than transmit diversity.

SINGLE-USER SPATIAL MULTIPLEXING

SU-MIMO

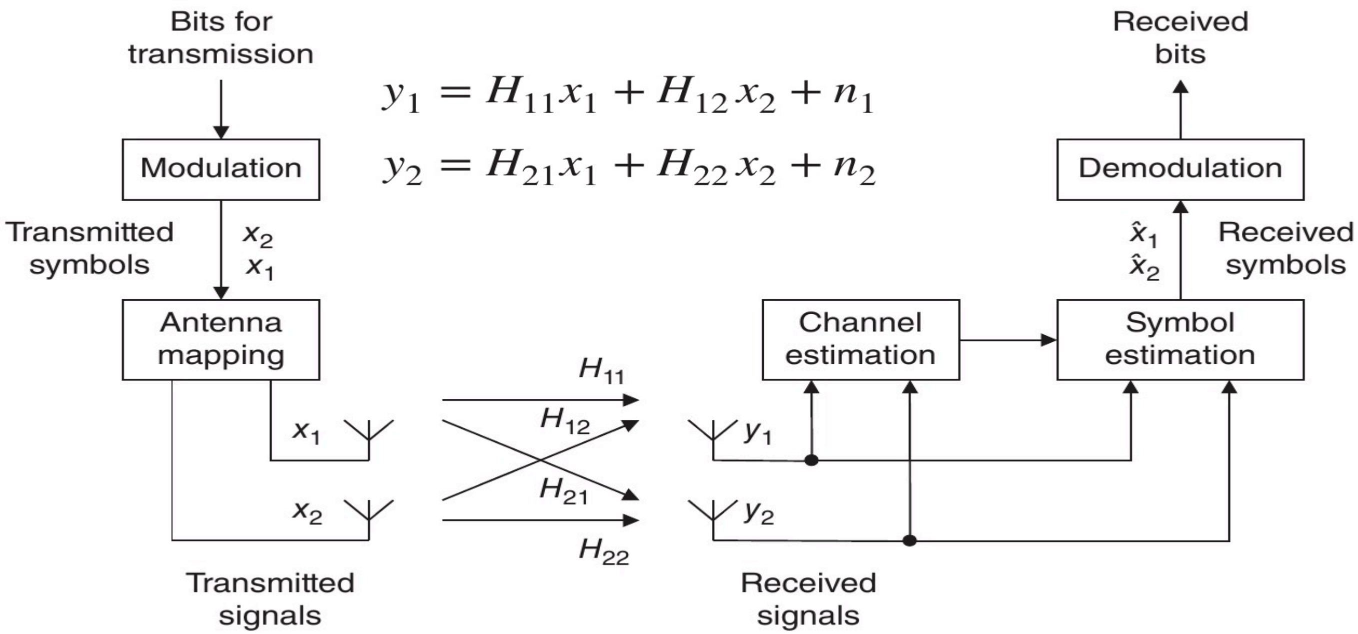
Concepts

- Antennas for improving data rate. Parallel, space separated, data streams towards the same user. Same frequency and time
- **Multipath is the key .** No multipath, no spatial multiplexing
- Multipath makes paths among antennas independent, thus channel matrix (*H*) can be inverted and signal demuxed
- Open-Loop and closed-loop implementations



Open-Loop

- N_T transmitting and N_R receiving antennas
- Max data rate equals to min(N_T, N_R) time the data rate of a single stream
- Do not require knowledge of the channel at the transmitter



Open-Loop

- In general, all the terms in the equations are complex.
  - *x,y,n* real and imaginary parts are the amplitudes of the in-phase and quadrature components
  - *h* module represents the amplitude attenuation introduced by the channel, and the h phase represents the phase shift
- Assuming no channel phase shift and BPSK, a possible case is:

$$
\begin{array}{llll}
H_{11} = 0.8 & H_{12} = 0.6 & x_1 = +1 & n_1 = +0.02 \\
H_{21} = 0.2 & H_{22} = 0.4 & x_2 = -1 & n_2 = -0.02
\end{array}
$$

- At the receiver we have

$$
\begin{aligned}
y_1 &= +0.22 \\
y_2 &= -0.22
\end{aligned}
$$

Open-Loop

- The receiver's first task is to estimate the four channel elements H_{ij} using reference symbols broadcasted by the transmitter during different times
- Actually, the estimated values contains noise and other errors thus the channel matrix seen by the receiver is $\widehat{H}$
- **Zero-forcing detector**: ignore noise in equations and invert the matrix to get x values. Modulation and coding have to solve the noise problem

$$\hat{x}_1 = \frac{\widehat{H}_{22}y_1 - \widehat{H}_{12}y_2}{\widehat{H}_{11}\widehat{H}_{22} - \widehat{H}_{21}\widehat{H}_{12}} \qquad \hat{x}_2 = \frac{\widehat{H}_{11}y_2 - \widehat{H}_{21}y_1}{\widehat{H}_{11}\widehat{H}_{22} - \widehat{H}_{21}\widehat{H}_{12}} \;\Rightarrow\; \hat{x}_1 = +1.1 \qquad \hat{x}_2 = -1.1$$

- Optimal detection can be achieved with **Maximum Likelihood** (ML) criterion. In most cases, ML can be implemented by finding the transmitted signal vector x that minimizes the Euclidean distance with respect to the received signal vector *y*

$$\hat{x} = \arg\min \| y - Hx \|^2$$

Open-Loop : issues

$$\boldsymbol{y} = \boldsymbol{H}.\boldsymbol{x} + \boldsymbol{n}$$

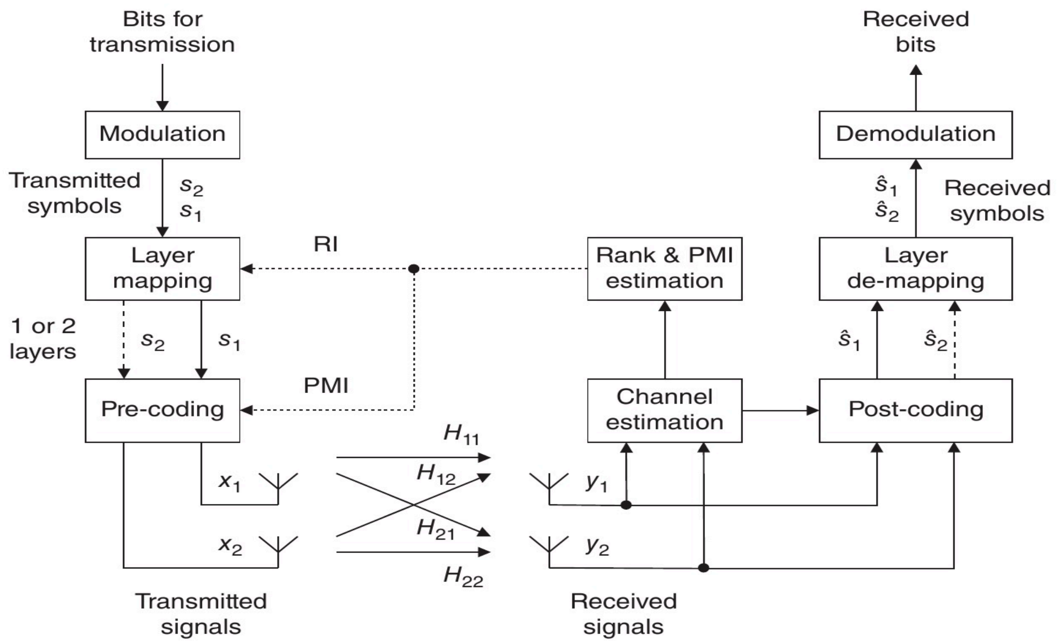
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\hat{\boldsymbol{x}} = \hat{\boldsymbol{H}}^{-1} . \boldsymbol{y}$$

- If number of transmitting antenna (N_T) is different by the number of receiving ones (N_R), the *H* matrix is not square and inversion is not possible
- If paths are correlated det=0 or close to zero, inversion is not possible or leads to many decoding errors
- In these cases closed-loop techniques help

Closed-Loop

- Transmitter knowledge of the channel is used to “pre-code” the tx symbols to improve paths’ isolation
- Pre-coding weights the symbols sent on the different antennas and weighs are represented by a Pre-coding Matrix
- The Pre-coding Matrix must by computed by receiver using reference symbols and it is communicated back with Pre-coding Matrix Indications (PMI) signaling messages
- Pre-coding Matrix computation requires knowledges on Shannon capacity for MIMO systems
- Eventually, the achieved capacity of closed loop system may get close to this bound

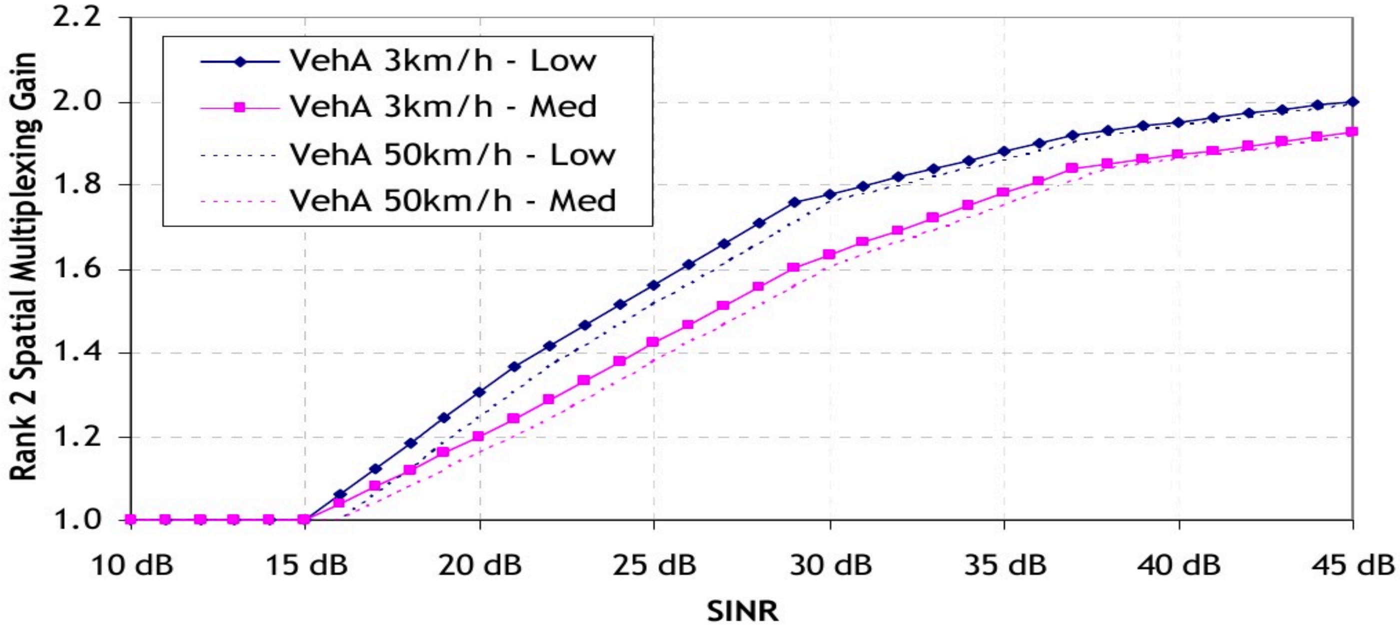


$$C = \sum_{k=1}^{K} \log_2\left(1 + \frac{P_k}{N}\sigma_k^2\right) \text{(bit/s/Hz)}$$

Attenuation of the eigenmode k-th of the channel (SVD)

GAINs

- Throughput gain with respect to SISO (tx=1, rx=1)
- Different speed, and different channel correlation level



NOTES ON SHANNON CAPACITY FOR SPATIAL MULTIPLEXING

GESBERT, DAVID, ET AL. "FROM THEORY TO PRACTICE: AN OVERVIEW OF MIMO SPACE-TIME CODED WIRELESS SYSTEMS." SELECTED AREAS IN COMMUNICATIONS, IEEE JOURNAL ON 21.3 (2003): 281-302.
SEE ALSO HTTPS://WWW.ECE.NUS.EDU.SG/STFPAGE/ELEZHANG/LECTURE_NOTES/LECTURE4_2010.PDF

Singular Value Decomposition

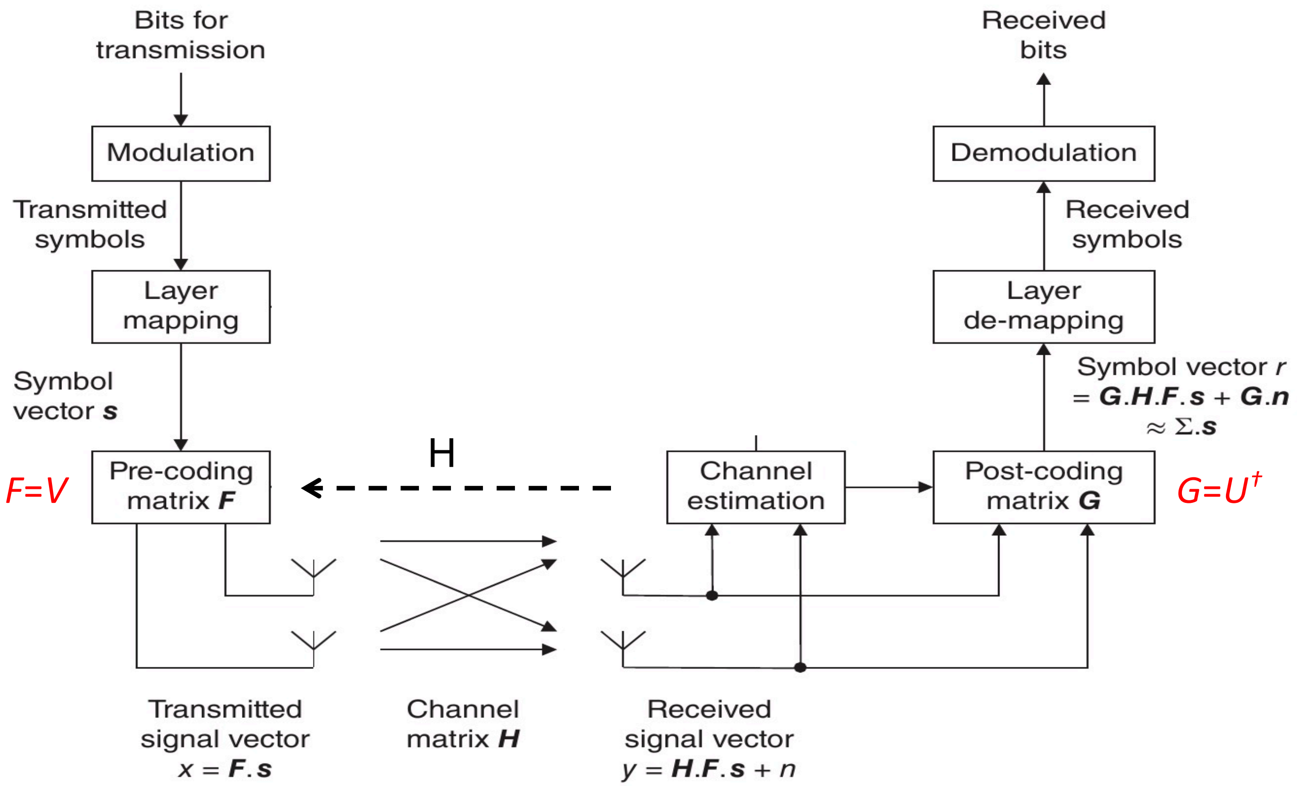
- Singular value decomposition (SVD) of the channel matrix

$$\boldsymbol{H} = \boldsymbol{U}.\Sigma.\boldsymbol{V}^{\dagger} \qquad \Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

- U is a N_R x N_R matrix, V is a N_T x N_T matrix. V^T is the Hermitionan conjugate (complex conjugate of the transpose matrix)
- U and V are unitary matrix, i.e. $U U^{\dagger} = I$ and $V V^{\dagger} = I$
- Σ is a diagonal rectangular matrix (N_R x N_T), whose values are the square roots of the eigenvalues of $HH^{\dagger}$, and are the singular values of H

Singular Value Decomposition

- Let us assume to apply a pre-coding matrix *F=V*. And a post-coding matrix *G=U†*



Singular Value Decomposition

- The received signal after post-coding is $\boldsymbol{r} = \boldsymbol{G}.\boldsymbol{H}.\boldsymbol{F}.\boldsymbol{s} + \boldsymbol{G}.\boldsymbol{n}$
- The the received symbol vector becomes the following in which we have independent symbols since Σ is diagonal:

$$\boldsymbol{r} \approx \boldsymbol{U}^{\dagger}.\boldsymbol{H}.\boldsymbol{V}.\boldsymbol{s} + \boldsymbol{V}.\boldsymbol{n}$$

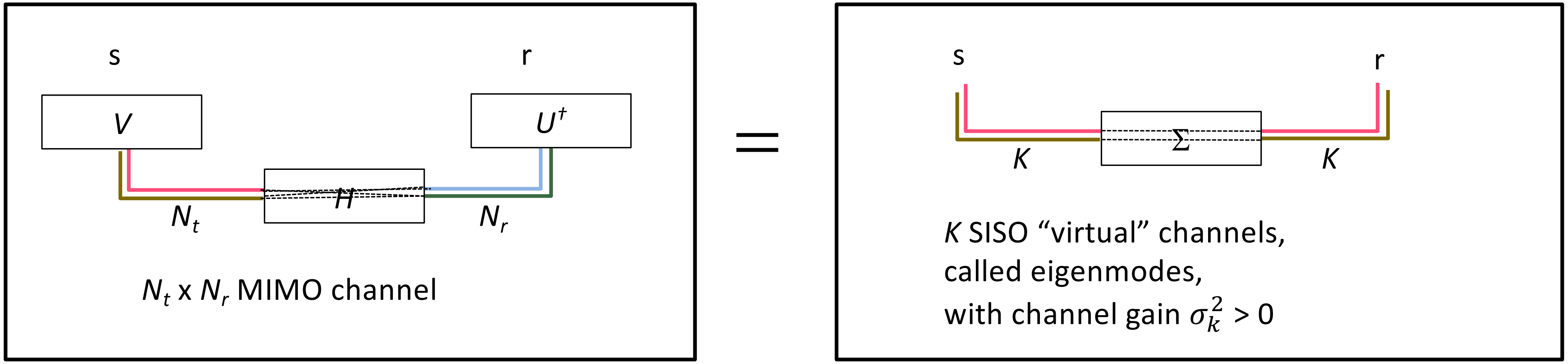
$$\approx \Sigma.\boldsymbol{s} + \boldsymbol{V}.\boldsymbol{n}$$

$$\xrightarrow[\text{noise}]{\text{Ignoring}} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \approx \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \Longrightarrow \hat{s}_i = \frac{r_i}{\sigma_i}$$

- Actually, by using SVD the MIMO channel is transformed into parallel SISO sub-channels with non-equal σ_i gains. Pre/post-coding has isolated the paths
- The resulting capacity is the sum of the Shannon capacity of the SISO sub-channels

Shannon Capacity

- Note: multiplying for unitary matrices does not alter the capacity of a channel
- Shannon capacity of a system with Pre/post coding is equal to that of a system without them



$$C = \sum_{k=1}^{K} \log_2\left(1 + \frac{P_k}{N}\sigma_k^2\right) \text{ (bit/s/Hz)}$$

SNR_k

P_k power tx on eigenmode k
K rank of the matrix H

Singular Value Decomposition for not symmetric antenna system

- If matrix H was singular (det=0) then the matrix Σ would have some diagonal value equals to zero

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \approx \begin{bmatrix} \sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

- We can use the SVD technique in exactly the same way if the numbers of transmit and receive antennas are different. With four transmit and two receive antennas, for example:

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \approx \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

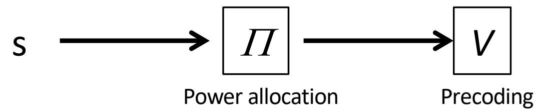
- The number of diagonal values of Σ different from zero (*H* rank) is the number of symbols that con be concurrently transferred

Eigenmode Transmissions

- How to put power on eigenmodes?
- The whole power of the transmitted symbols is limited to a power budget P
- Let us assume that the average power of the symbols s_i entering the pre-coding V matrix is unitary
- Multiplying power unitary symbols for a value $sqrt(p_i)$ the resulting average power is equal to p_i.

Eigenmode Transmission

- To allot an amount of energy $P = \{P_1, P_2, \ldots P_K\}$ over the sub-channels it is enough to multiply the normalized unitary pre-coding matrix V for a matrix Π with equal size of Σ, whose diagonal values are sqrt(P_i)



- Π is called “power allocation matrix”

$$r = U^h \left(U\,\Sigma\,V^h\right) V\,\Pi\,s \;=\; \Sigma\,\Pi\,s$$

- Power of element of s are unitary thus power of elements of r is exactly $P_k \sigma_k^2$

$$C = \sum_{k=1}^{K} \log_2\left(1 + \frac{P_k}{N}\sigma_k^2\right) \text{ (bit/s/Hz)}$$

Optimal power allocation: Water filling

- How to distribute a budget of *P* power over *k* eigenmodes to maximize capacity?
- Shanon bound computation actually requires also this optimal choice
- Idea: since power is within Log, it is better to give power to eigenmodes with small σ_k^2

Problem: $$\boxed{\max_{P_k, \sum_{k=1}^{n} P_k \leq P} \sum_{k=1}^{n} \log_2\left(1 + \frac{P_k}{N}\sigma_k^2\right)}$$

Solution: (need solve for μ)

$$C = \sum_{k=1}^{n} \log_2\left(1 + \frac{P_k^* \sigma_k^2}{N}\right)$$

$$P_k^* = \left(\mu - \frac{N}{\sigma_k^2}\right)^+, \qquad \sum_{k=1}^{n} P_k^* = P$$

$X^+ = \max(x,0)$

Uniform Power Allocation

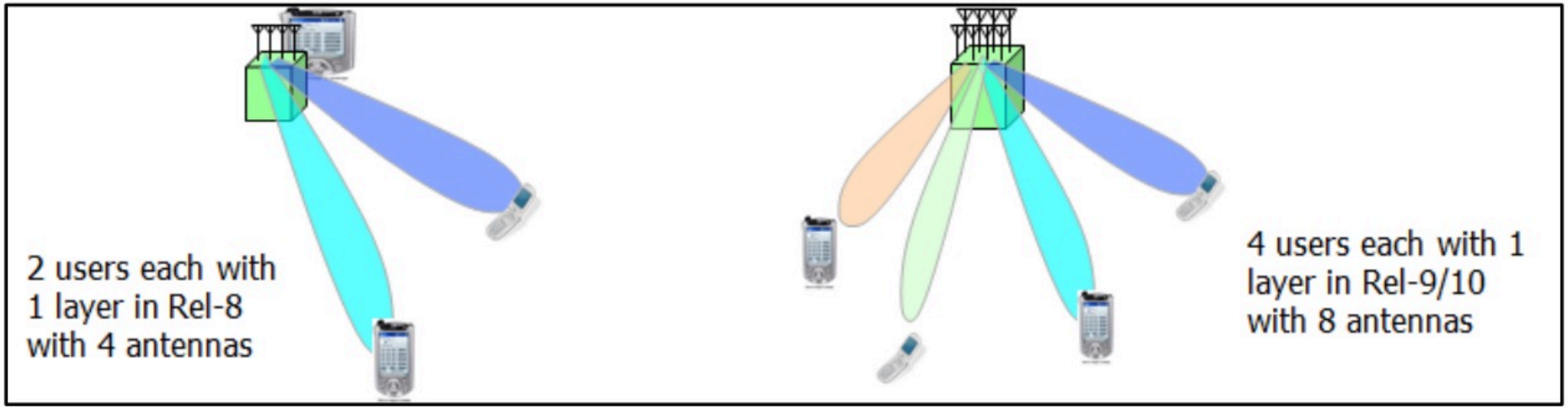
- The transmitter applies a uniform power allocation matrix formed by a number of elements equals to the number of transmitting antennas
- As a result on each eigenmode there is a power equals to $P_k^* \cong P_T/N_t$
- For high SNR (and full rank), $\mu >> N/ \sigma_k^2$
- Thus, simpler uniform power allocation solution is close to the water filling

BEAMFORMING

SPATIAL FILTERING, MU-MIMO

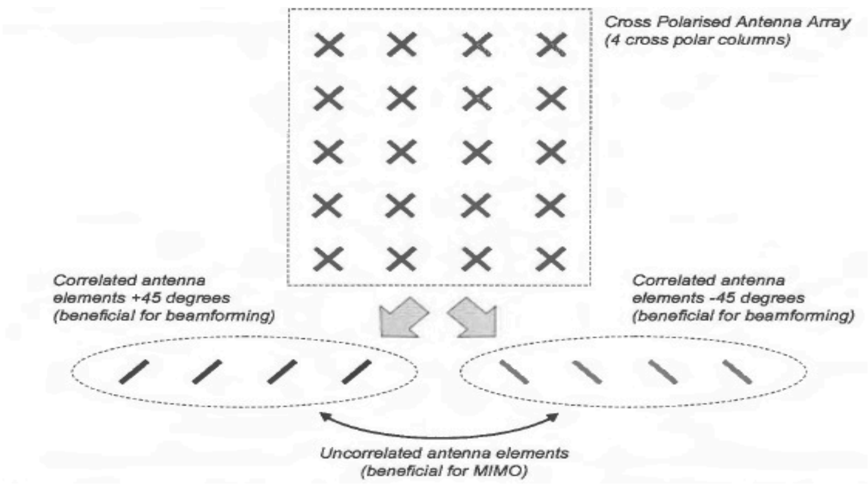
Principles

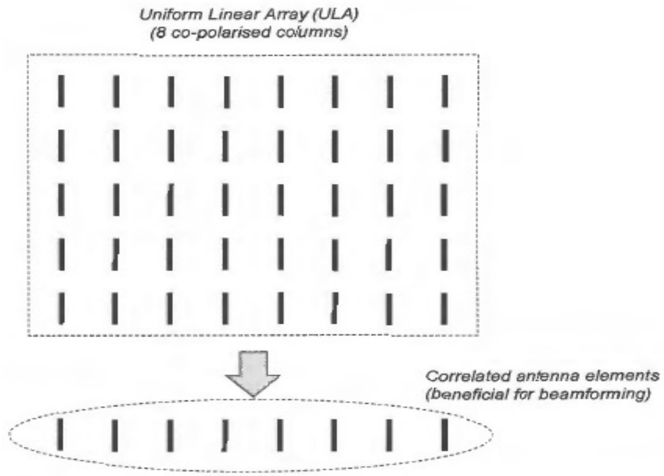
- *To create different beams for different users*
- *Partitioning of the space between BS and user*
- *Parallel transmissions towards different users*
- *Dramatic improvement of cell capacity*
- *Can be combined with SU-MIMO (e.g., 4 beams towards a same user is similar to SU-MIMO with Nt=4)*



Antenna

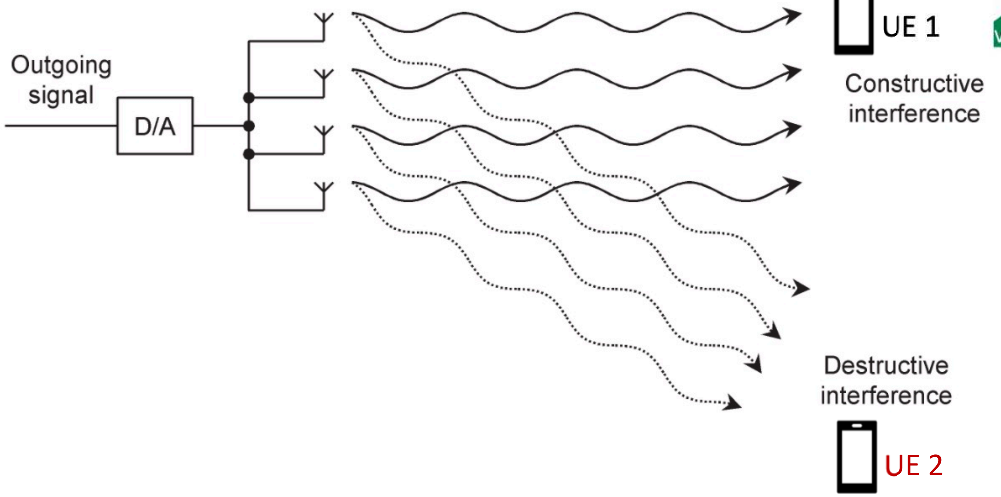
- *Antennas used for beamforming usually have several antenna elements separated half of a wavelength*
- *Requires signals coming from the different antenna elements must be high correlated, i.e., little multipath and line of sight*
  - *Completely different requirement wrt diversity and spatial multiplexing systems*
  - *May requires different antenna elements*



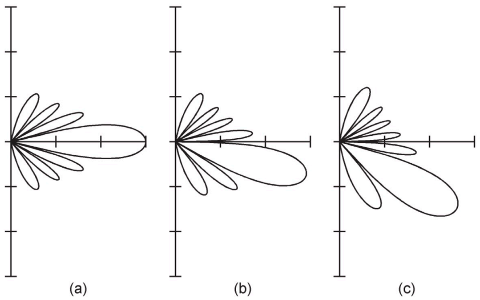


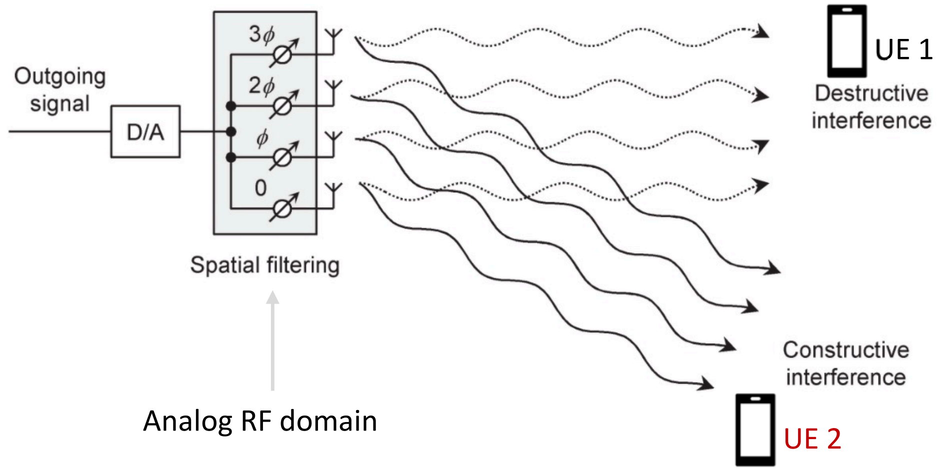
Spatial Filtering with Analog Beamforming

- *Mobile 1 is at right angles to the antenna array. Constructive interference, signal power gain*
- *On the other hand, mobile 2 is at an oblique angle and receives signals from alternate antennas that are 180∘ out of phase. These signals interfere destructively, so the received signal power is low*
- *Applying a phase ramp Φ to analog signal we can change the beam direction towards θ_0*



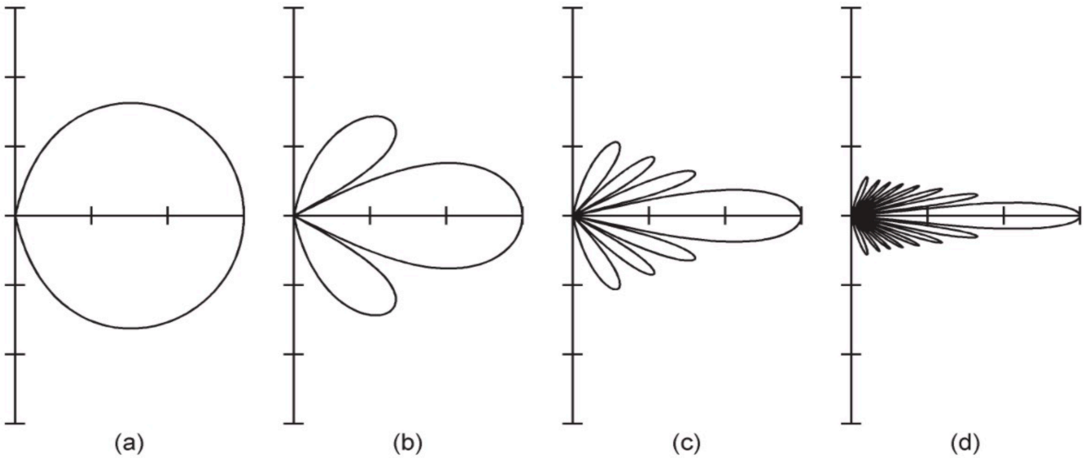
$$\phi = \frac{2\pi d \sin\theta_0}{\lambda}$$





Beams characteristics

- *More antennas, narrower beams*
- *Angle between the main lobe and the first null is roughly* $\frac{\lambda}{Md}$
  - *M number of antennas*
  - *d distance*
  - *λ wavelength*



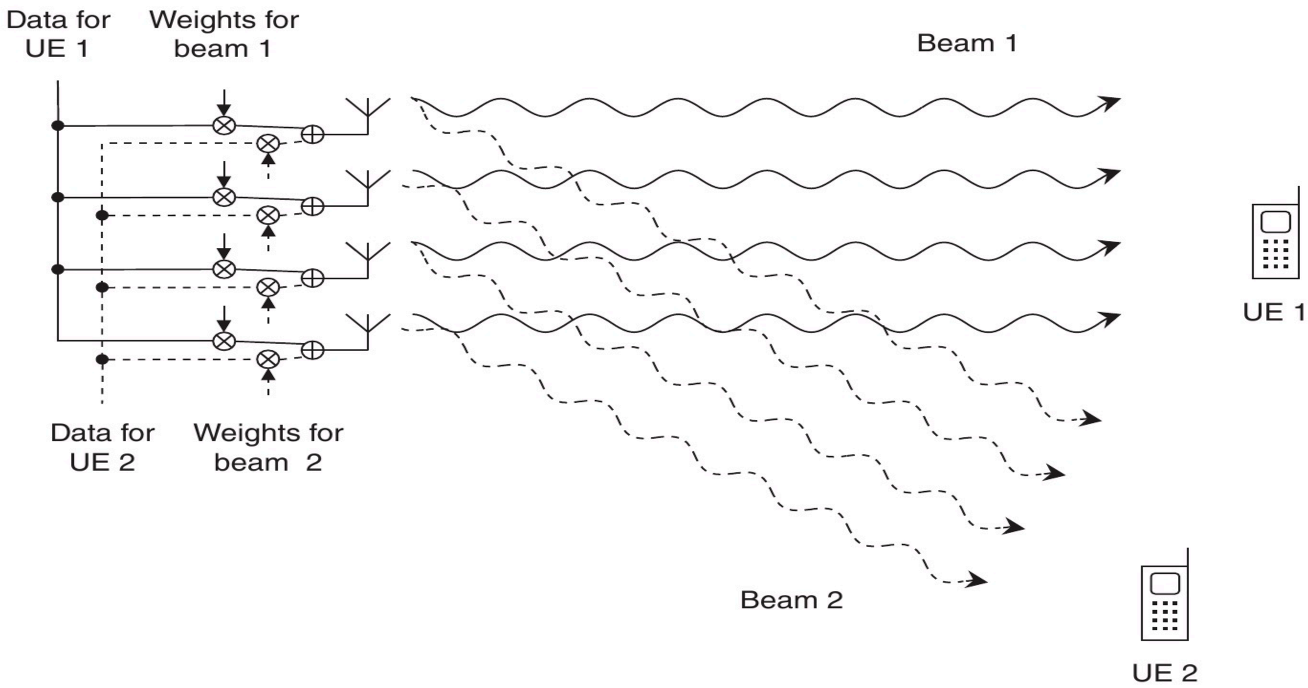
2 antennas 4 antennas 8 antennas 16 antennas

Spatial Filtering (Analog Beamforming) – Pros and Cons

- *Pro:*
  - *The signal phases of individual antenna signals are adjusted in RF domain, i.e., after a single D/A conversion.* ***Simplicity*** *and* ***low power consumption***
  - *Impacts the radiation pattern and gain of the antenna array, thus* ***improves coverage***
  - *Overcomes partly the impact of high pathloss in mmWave*
- *Cons:*
  - *Only one beam per set of antenna elements can be formed*
  - *Static beams, each covering a zone of territory, similar to electric sectoring/down-tilting*
  - *Lower capacity improvement with respect to no MU-MIMO, for which beams are per-user*

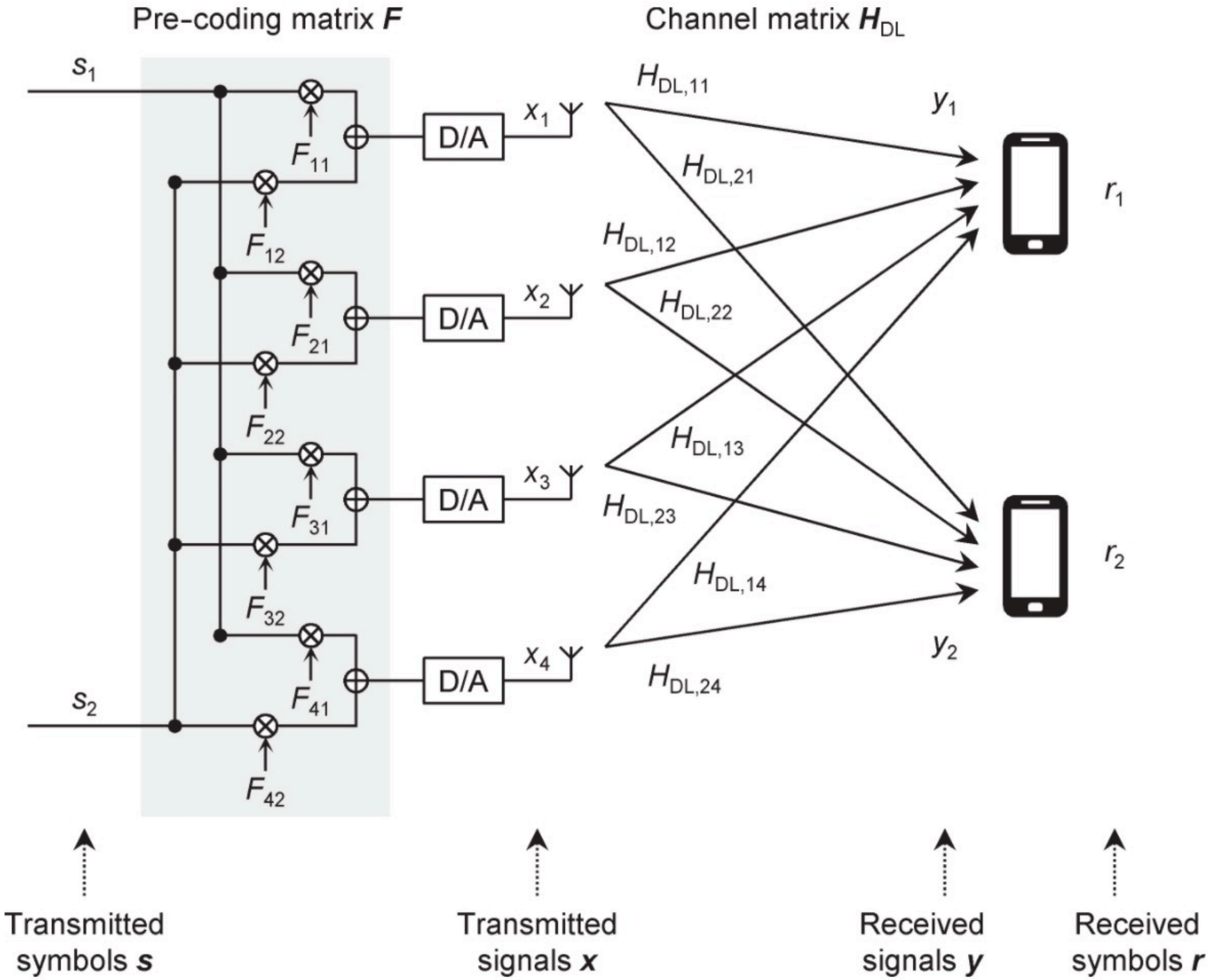
MU-MIMO with Digital Beamforming

- *Multiple beams from the same antenna set*
- *Different complex weights (amplitude/phase) applied to different data streams of different users*
  - *The weights applied to the data stream for UE1 provide a null on UE2 and viceversa*



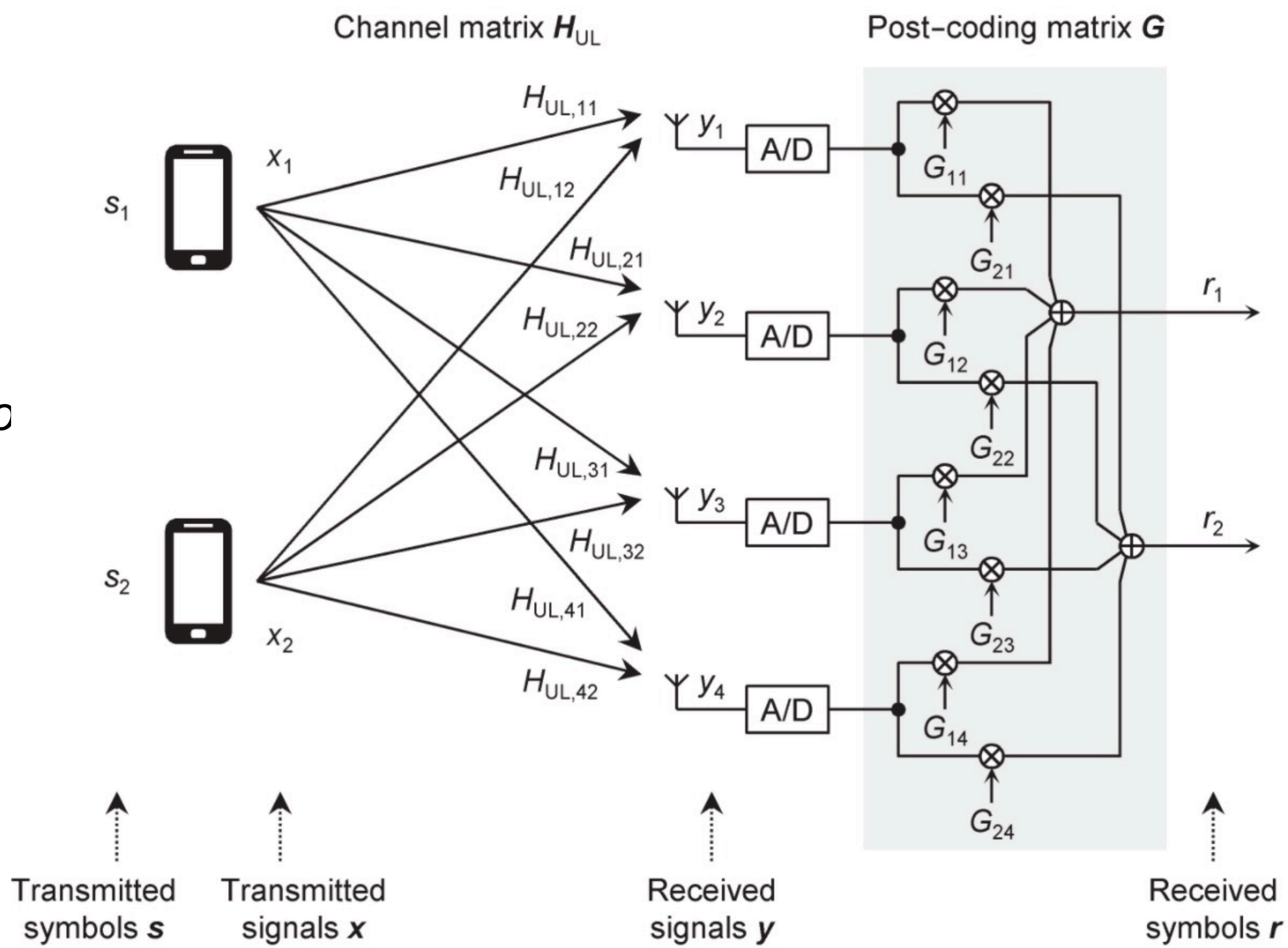
MU-MIMO Downlink with Digital Beamforming

- *Implementation via Digital Beamforming*
  - *Processing made in digital baseband,* ***before D/A***
- *Assuming to have M=4 antennas, UE_x uses a set of M complex weights (F_{yx}) applied to its symbols to create a beam main lobe in UE_x direction and nulls in the directions of other UEs concurrently served*
- *Many per-user beams through the same antenna set*
- *Matrix F (simply) computed by gNB via reference signals sent by all UEs*
  - *channel reciprocity TDD*



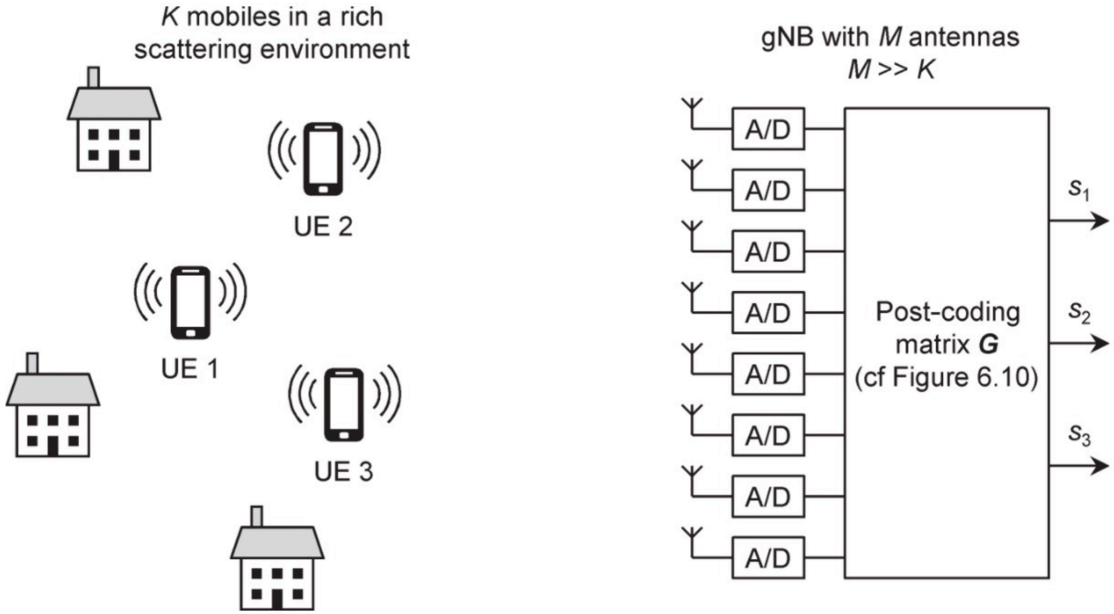
MU-MIMO Uplink with Digital Beamforming

- *Similar to dowlink but does't requires channel reciprocity to estimate weights (G)*



Massive MIMO (mMIMO)

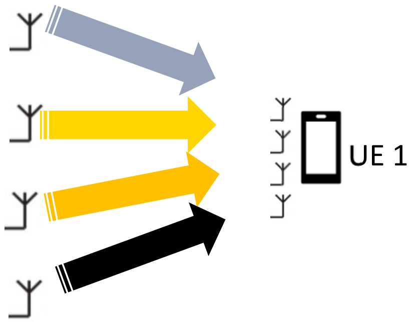
- *MU MIMO relies on placing interfering mobiles either in the nulls of the base station's transmission and reception beams, or very close to them.*
  - *With many mobiles it becomes increasingly difficult to place all the nulls in the correct directions: instead, the interference levels rise, and the technique quickly becomes unusable*
- *Solution M >> K, that is* ***massive MIMO***



SU-MIMO, Spatial Filtering, MU-MIMO

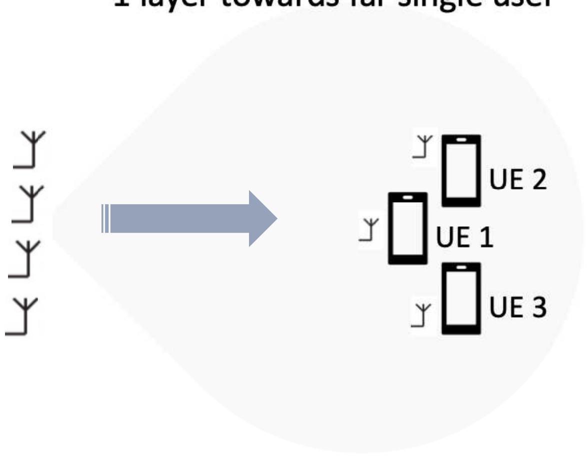
SU-MIMO

4 layers towards a close single user



Spatial Filtering

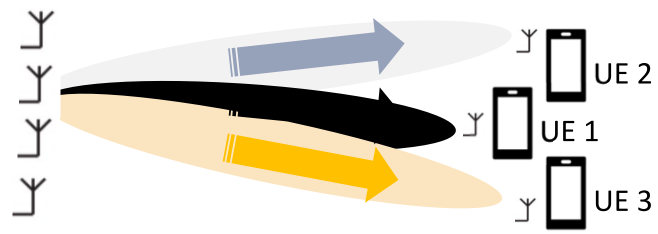
1 layer towards far single user



Fixed beam

MU-MIMO

3 layer towards different single users
1 layer per user



Beams stearing

MIMO Decision Tree