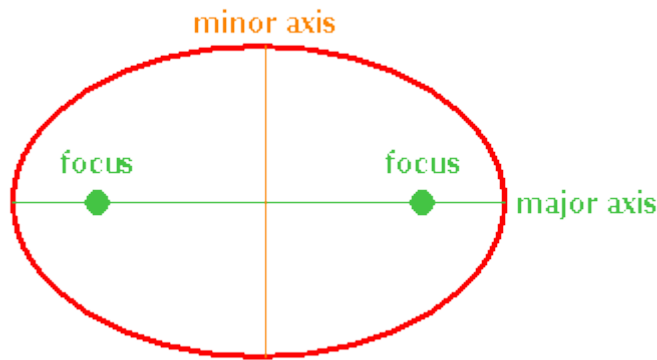


Kepler's Laws (1609)

1. The orbit of a satellite is
an ellipse
with the Earth at one of the two foci



$2a$ = major axis

$2b$ = minor axis

eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

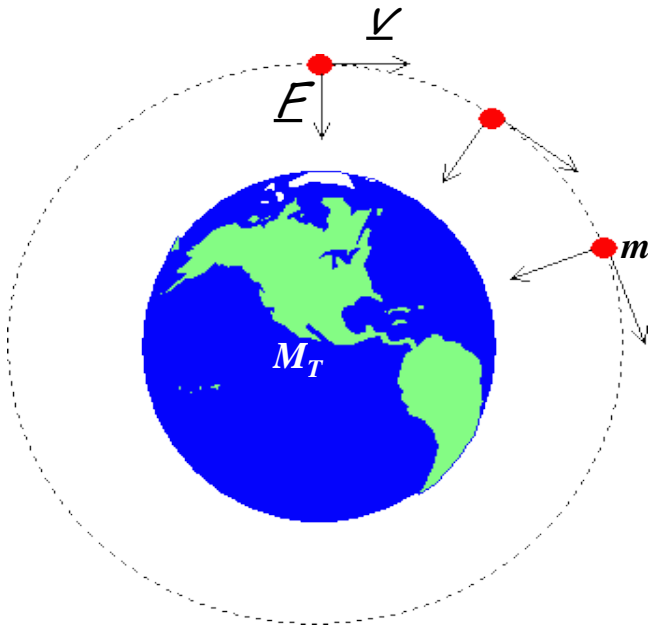
$$0 < e < 1$$

$$e = 0 \text{ circle}$$

$$e = 1 \text{ parabola}$$

$$e > 1 \text{ hyperbola}$$

Forces acting on a satellite



\underline{E} is the gravitational force
 \underline{v} is the tangential velocity

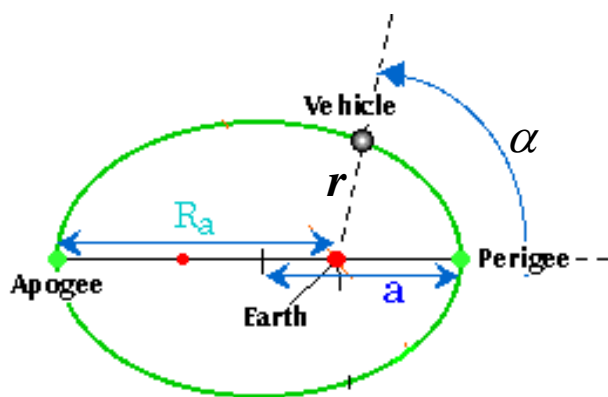
$$-\frac{GM_T m \hat{r}}{r^2} = m \ddot{\underline{r}}$$

\underline{r} is the vector from the centre of the Earth to the Satellite

\hat{r} is the unit vector $= \frac{\underline{r}}{|\underline{r}|}$

In the absence of other forces,
 satellite position along the ellipse is:

$$r = \frac{a(1 - e^2)}{1 + e \cdot \cos \alpha}$$



R_A = distance Earth-Apogee

R_P = distance Earth-Perigee

α = anomaly

$\alpha(P) = 0^\circ$

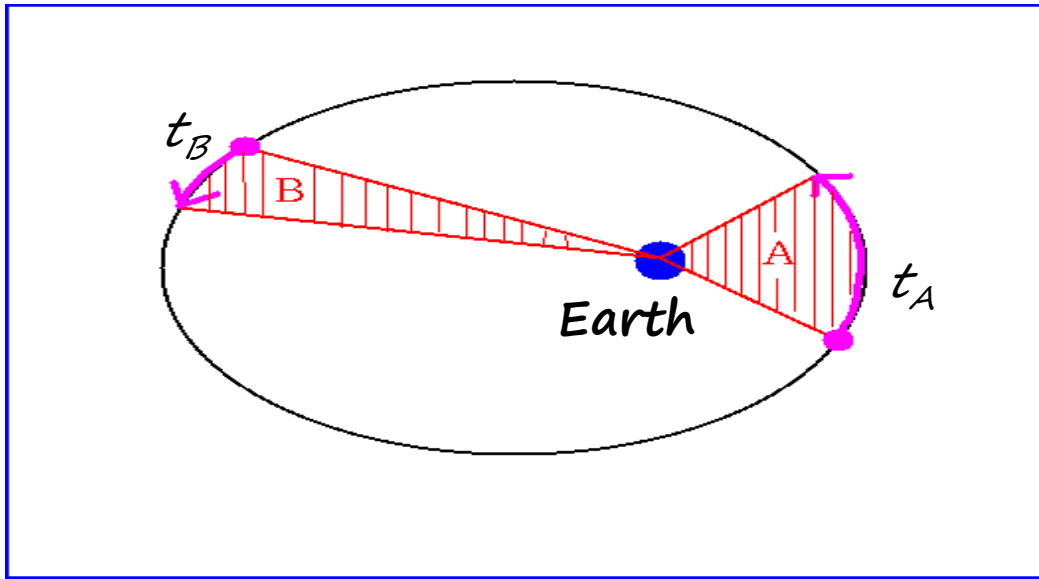
$\alpha(A) = 180^\circ$

$$R_A = a(1 + e)$$

$$R_P = a(1 - e)$$

$$e = \frac{R_A - R_P}{2a}$$

II. The line segment joining the satellite and the Earth sweeps out equal areas during equal intervals of time



When $t_A = t_B$ then Area A = Area B

From energy conservation,
the orbital velocity at distance r is:

$$v^2 = \frac{\mu}{a} \left(\frac{2a}{r} - 1 \right) \quad (\text{with } \mu = G \cdot M_T)$$

$$v_P = \sqrt{\frac{2\mu R_A}{R_P(R_A + R_P)}} \quad \text{Velocity at Perigee}$$

$$v_A = v_P \frac{R_P}{R_A} \quad \text{Velocity at Apogee}$$

and $v_P > v_A$

III. The square of the orbital period of a satellite is proportional to the cube of the semi-major axis of its orbit

$$T^2 = \frac{4\pi^2}{\mu} a^3$$

For a circular orbit:

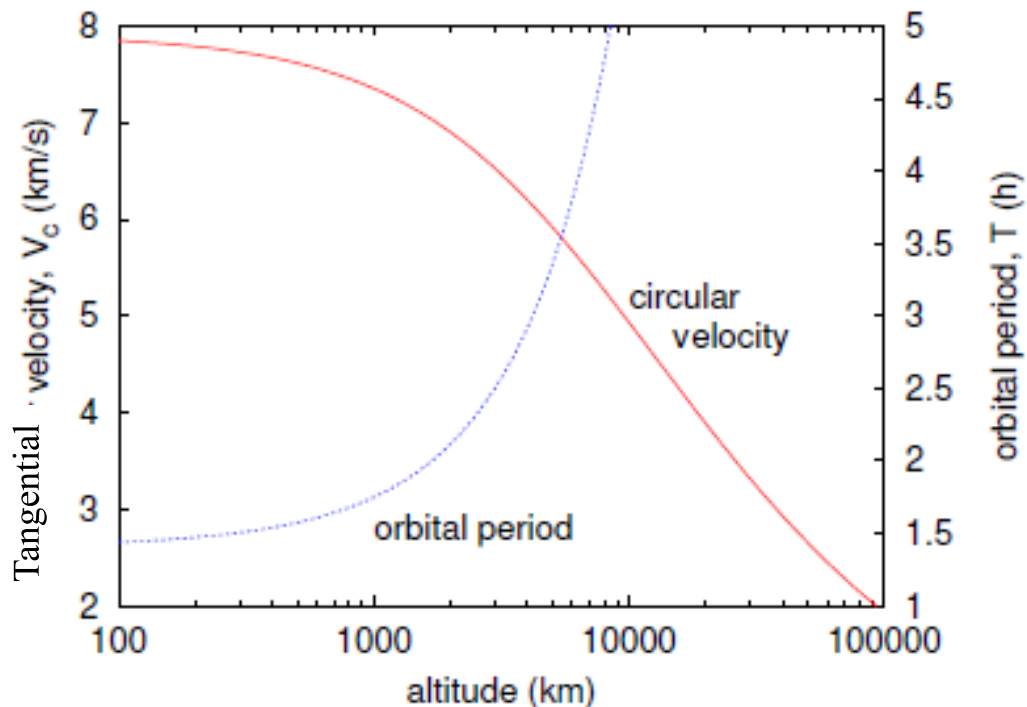
$$\frac{GM_T m}{r^2} = m \frac{v^2}{r}$$

Angular velocity: $\dot{\theta} = \frac{v}{r} = \sqrt{\frac{\mu}{r^3}}$

$$T = 2\pi \sqrt{\frac{r^3}{\mu}}$$

The satellite velocity and its orbital period depend only on its altitude (and on the Mass of the central body (Earth))

Tangential velocity and orbital period



Gravitational parameter of the Earth

$$\mu = GM_T = 398600 \text{ km}^3/\text{s}^2$$

Universal gravitational constant G

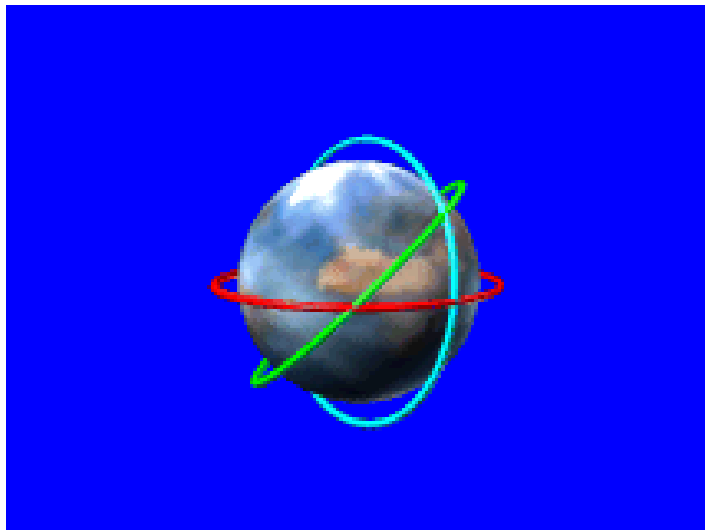
Mass of the Earth M_T

Mean Earth Radius

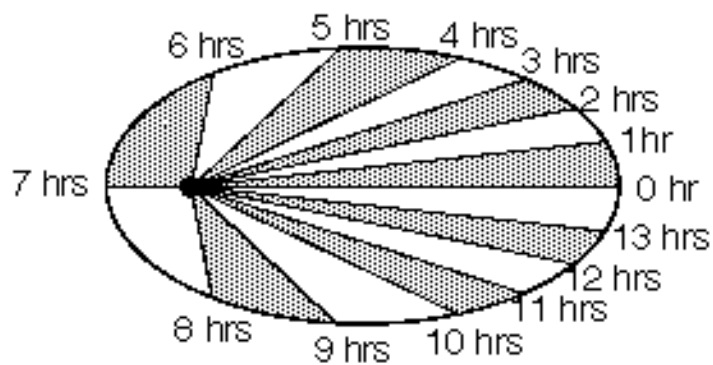
$$R_T = 6378 \text{ km}$$

Satellite altitude

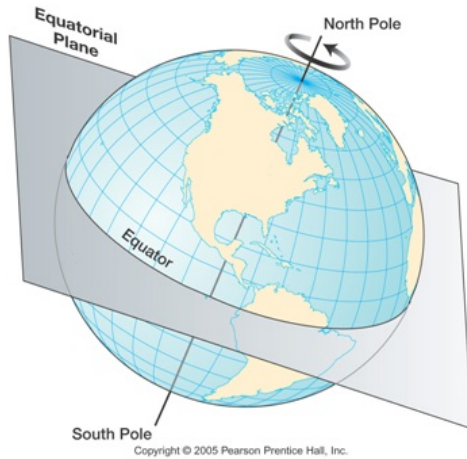
$$h = r - R_T$$



Time Versus Area Swept by an Orbit

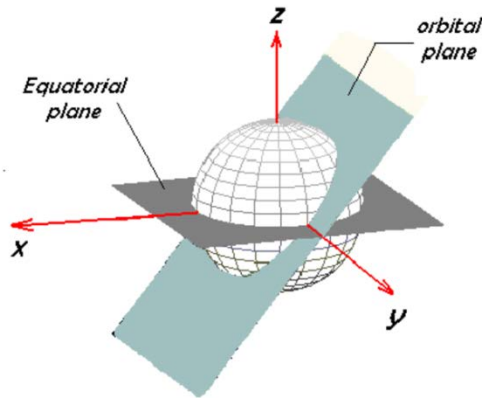


The equatorial plane



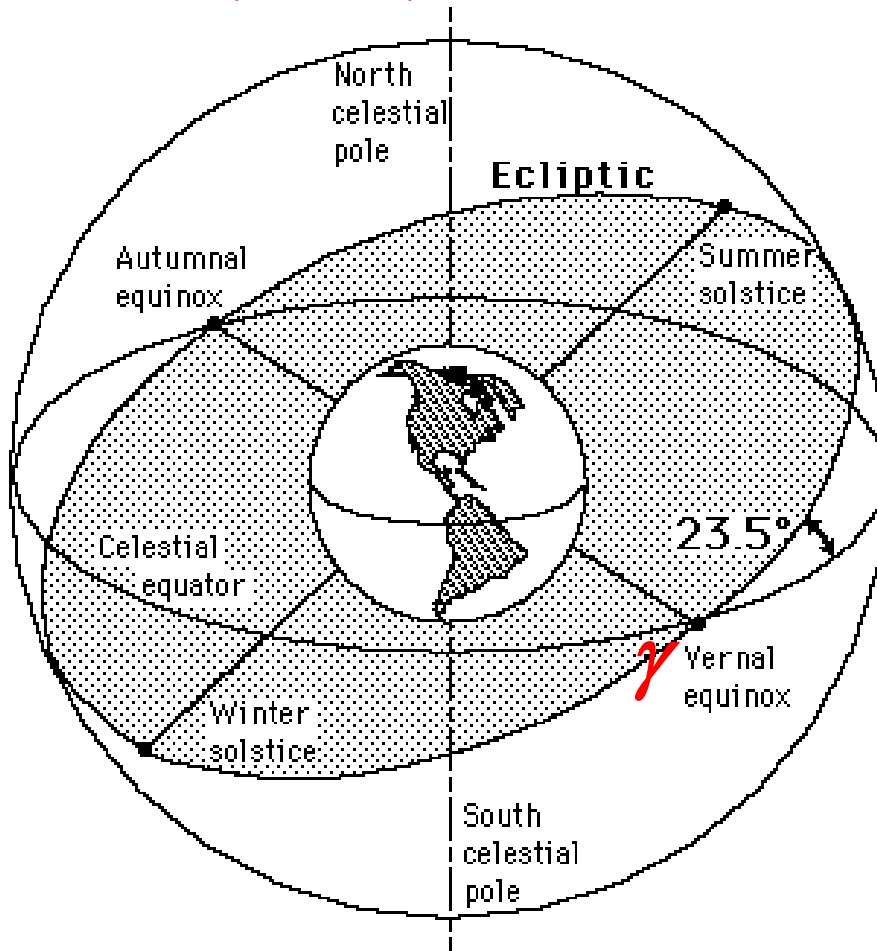
is the plane at 90°
with respect to the terrestrial axis
that contains the Earth's equator

The orbital plane



is the plane that contains the satellite orbit

The ecliptic plane



is the plane containing the
“Sun’s orbit around the Earth”

The Sun completes its orbit in one year

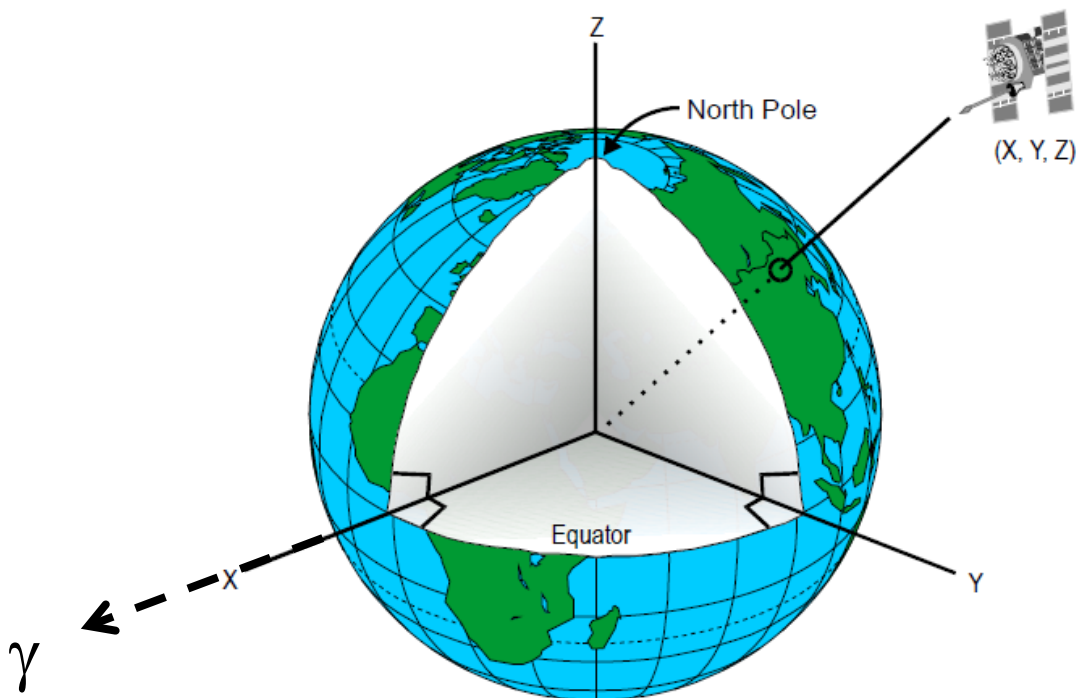
The First point of Aries (γ) lies in the
intersection between the equatorial plane
and the ecliptic plane
at the vernal equinox (~March 21st)

We introduce the following
inertial reference frame

Earth Centered equatorial system
ECI

The origin is at the center of the Earth

- x axis towards the first point of Aries γ
- z axis towards the North pole
- y axis completes a right-handed set



$$-\frac{GM_T m}{r^2} \hat{r} = m \frac{d^2 r}{dt^2} \hat{r}$$

It is a 2nd order
vector differential equation

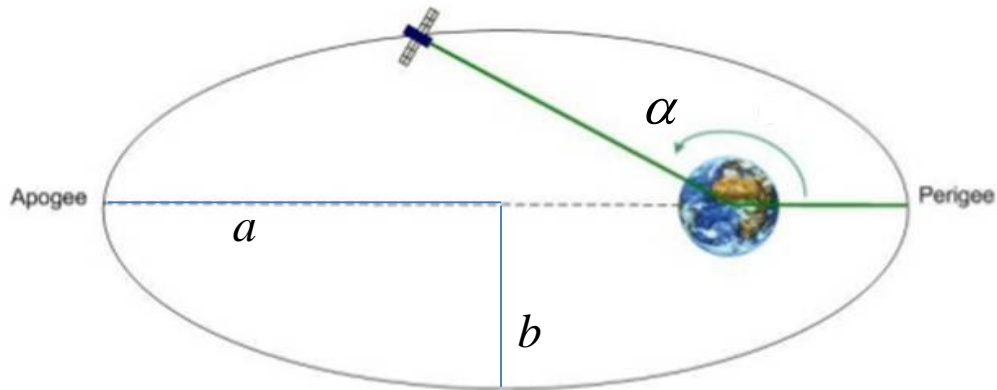
Its solution requires the knowledge of
6 quantities. In general, initial
position and initial velocity are given.
But they are not easy to depict

The orbital elements
(keplerian elements)

are used instead

The nomenclature is borrowed from
astronomy

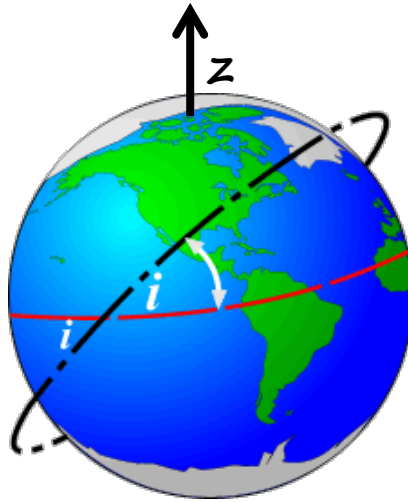
3 parameters allow positioning of the satellite along its orbit



- Semi-major axis: a
- Eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}}$
- Anomaly at time t_0 : $\alpha(t_0)$

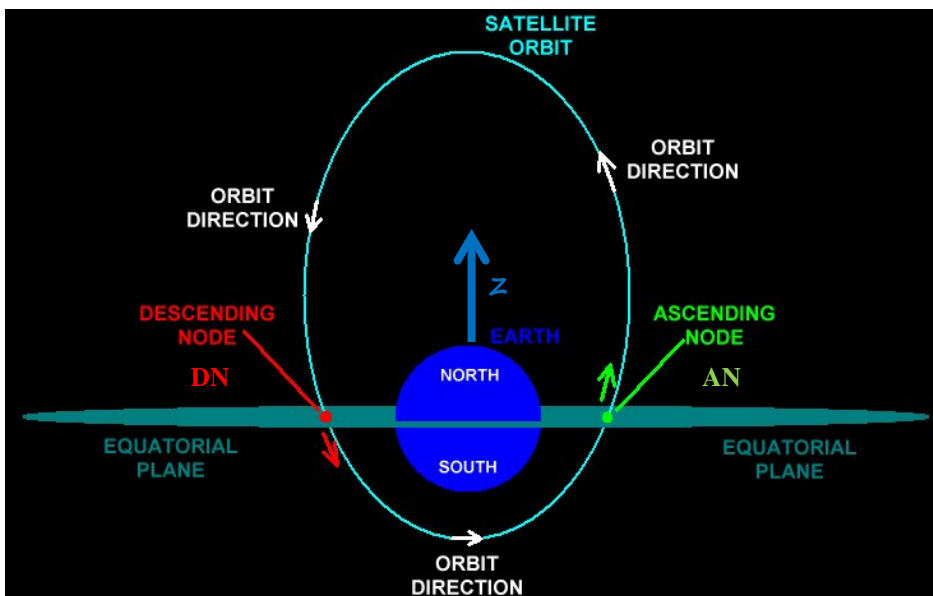
And 3 parameters allow positioning of the satellite in the sky:

Inclination i



Is the angle between the orbital plane and the equatorial plane

The intersection of these two planes defines the **line of Nodes**, that is the line connecting the **Ascending** and the **Descending** Nodes (where the satellite crosses the equator going towards the **North** Pole or **away** from it)

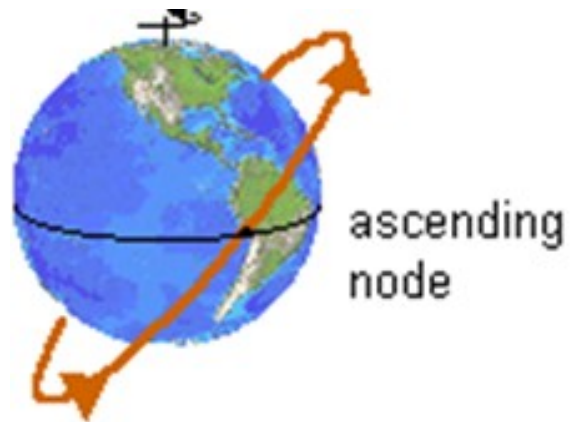


$$0^\circ < i < 180^\circ$$

but

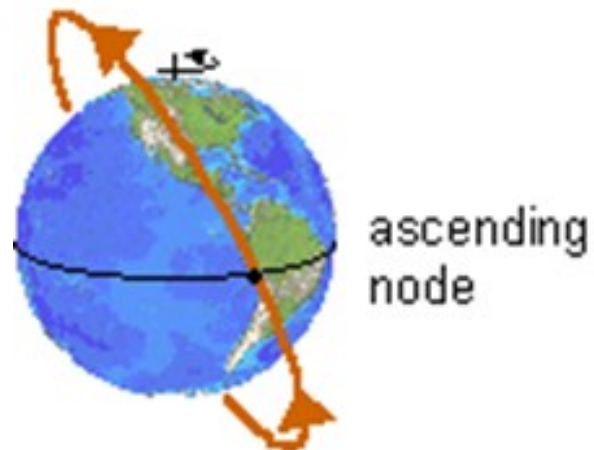
If the satellite rotates in the same direction of the Earth, the orbit is *direct* (prograde) and

$$0^\circ < i < 90^\circ$$



When the satellite moves against the direction of Earth's rotation, the orbit is *indirect* (retrograde) and

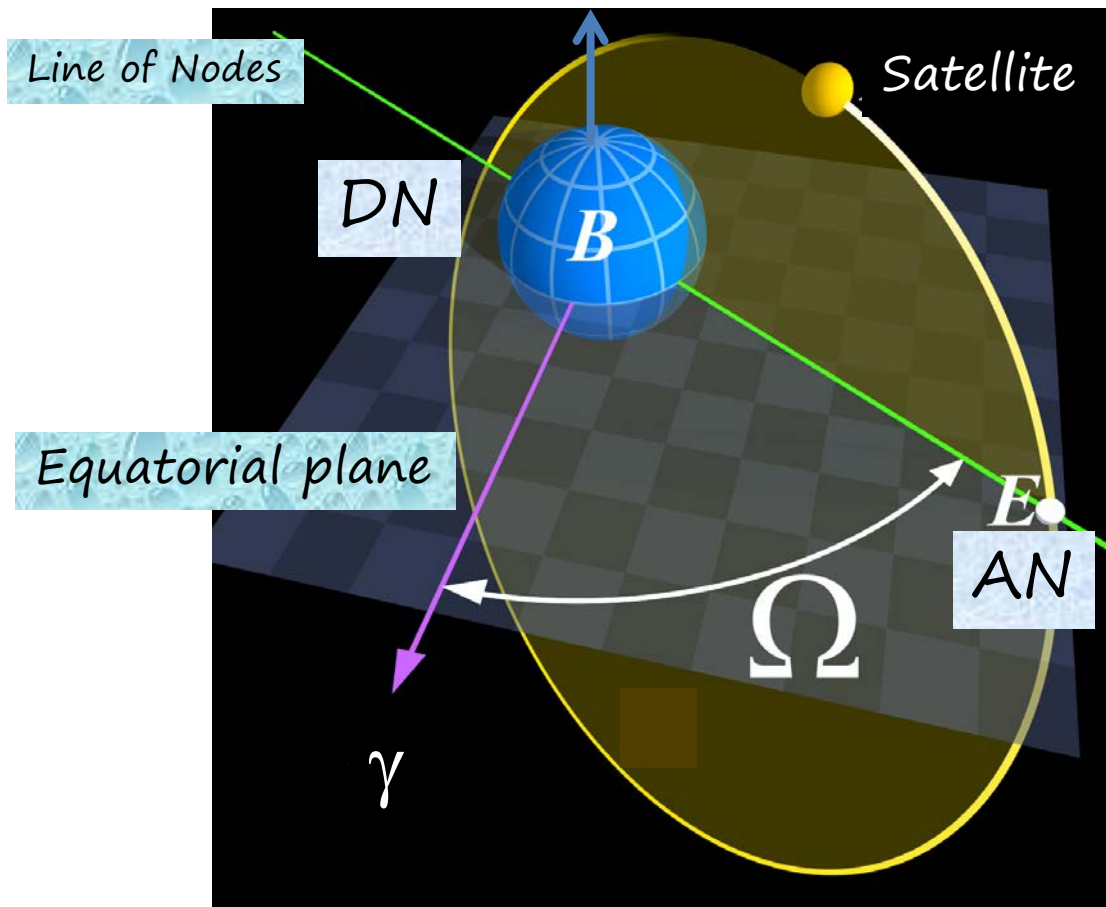
$$90^\circ < i < 180^\circ$$



Right Ascension of the Ascending Node Ω
(RAAN, longitude of the Ascending Node)

It is the angle between the line of Nodes
and the x-axis

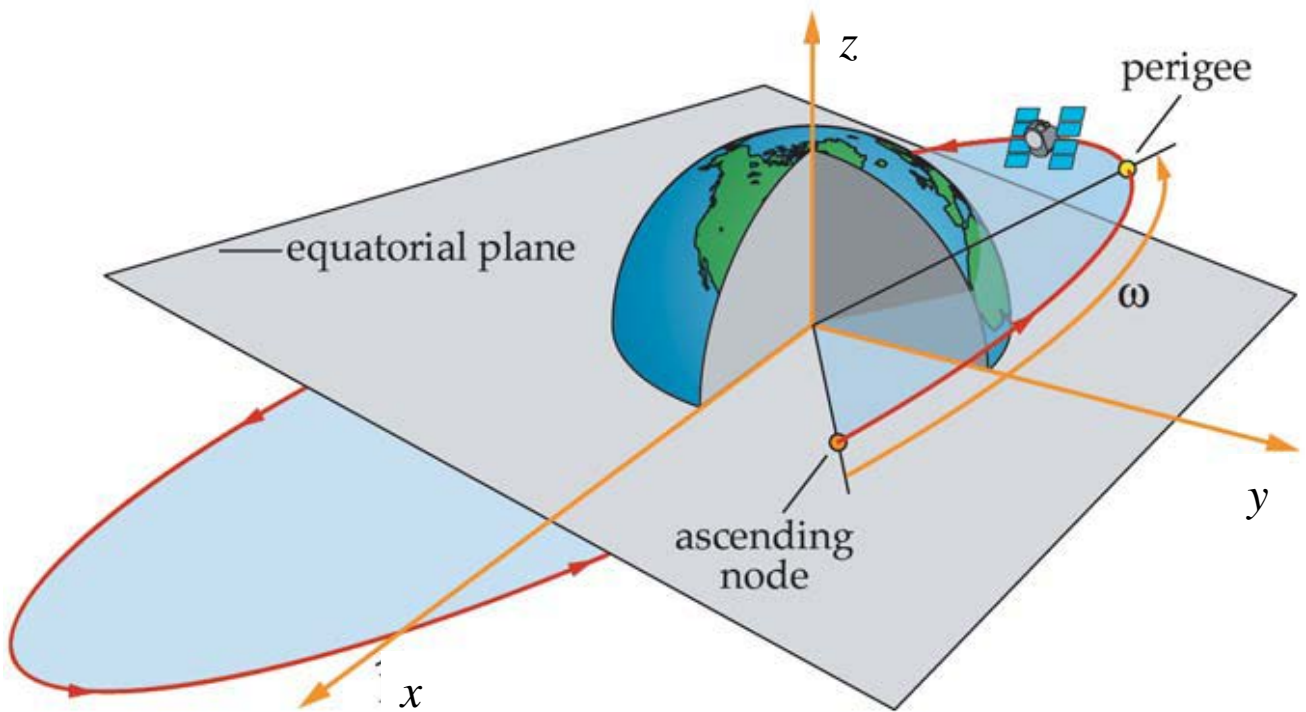
It lies in the equatorial plane
and it is measured eastward
towards the AN



$$0^\circ \leq \Omega \leq 360^\circ$$

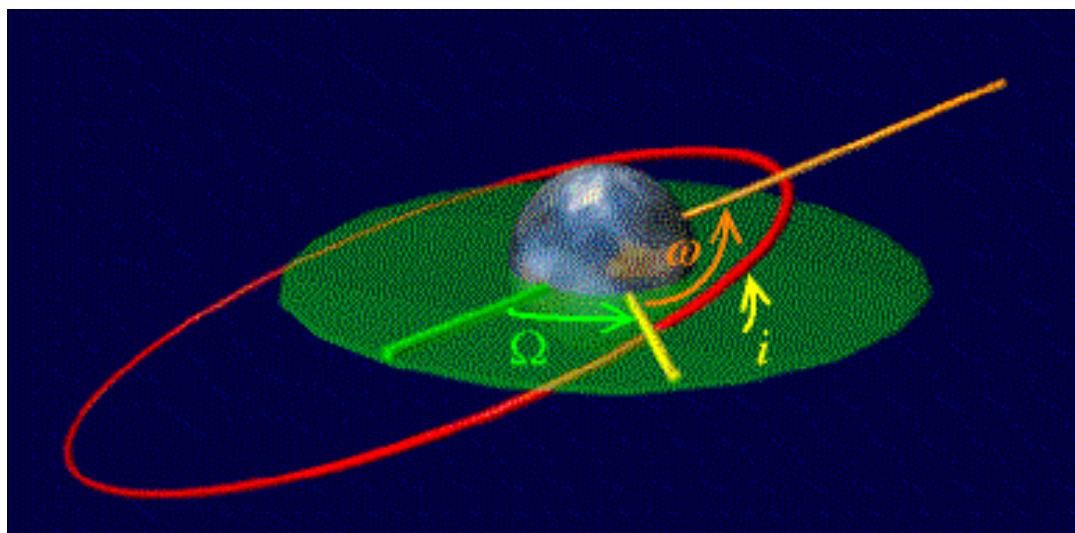
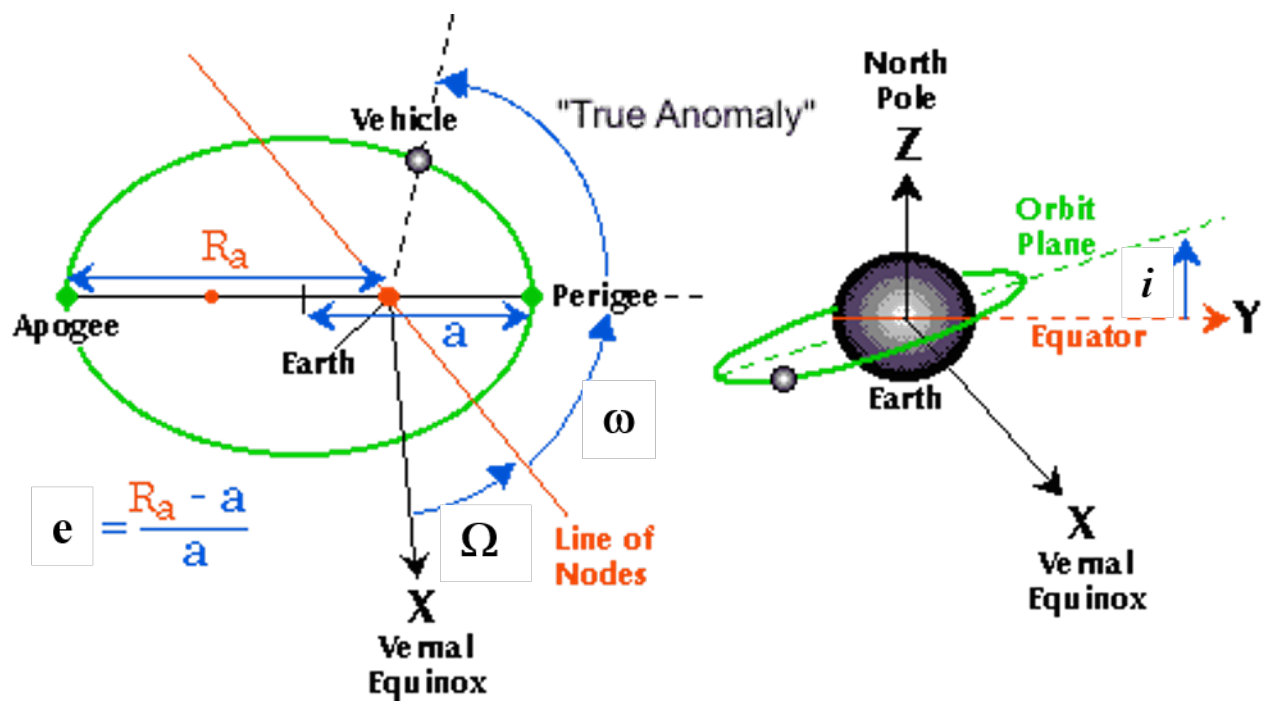
Argument of Perigee ω

Is the angle between the
Ascending Node and the Perigee



It is measured in the orbital plane and
it allows orientation of the orbit

$$0^\circ \leq \omega \leq 360^\circ$$



The geosynchronous orbit

The orbital period of a geosynchronous orbit is equal to the Earth rotational period

$$T = D_s = \text{sidereal day}$$

The solar day is measured wrt the Sun and it is ≈ 24 h

The sidereal day is measured wrt the fixed stars

$$D_s = 23\text{h } 56' 4'' = 86164.1 \text{ s}$$

$$D_s = 2\pi \sqrt{\frac{a^3}{\mu}}$$

$$a=42164 \text{ km}$$

*is the semi-major axis of
the geosynchronous orbit*

$$a=6.61 R_T$$

Geostationary orbit

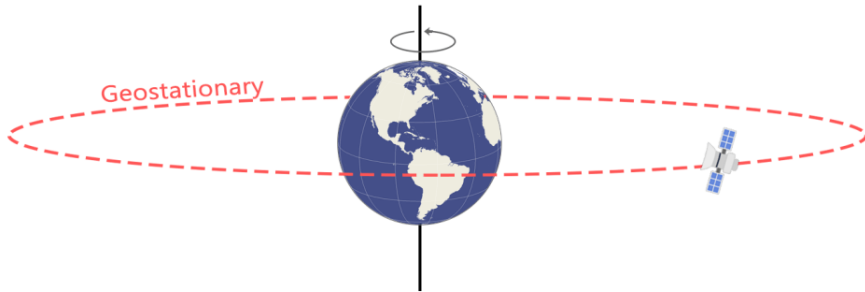
A geostationary satellite stays fixed above a specific location on Earth

- Circular orbit
- Equatorial orbit
- Prograde orbit
- $e=0$
- $i=0^\circ$
- Satellite moves eastward
wrt the North pole

$$h = 35786.103 \text{ km}$$

$$v = 3.07 \text{ km/s} = 11052 \text{ km/h}$$

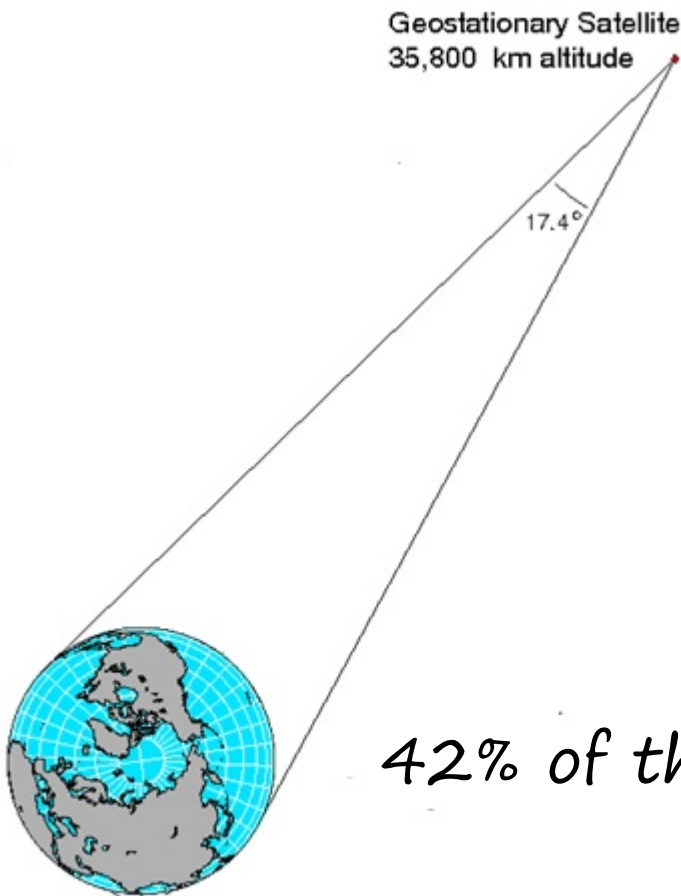
$$\dot{\vartheta} = 72.92 \cdot 10^{-6} \text{ rad/s}$$



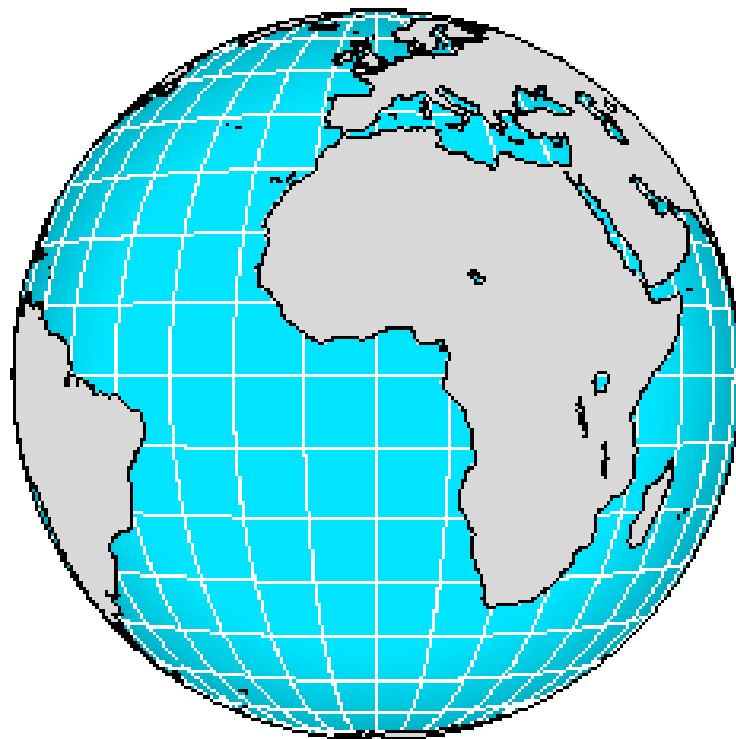
$$\underline{\dot{\mathcal{J}}} = \underline{\dot{\mathcal{J}}}_T$$

$$\underline{v} = \underline{\dot{\mathcal{J}}} \times \underline{r}$$

$$\underline{v}_T = \underline{\dot{\mathcal{J}}}_T \times \underline{R}_T$$

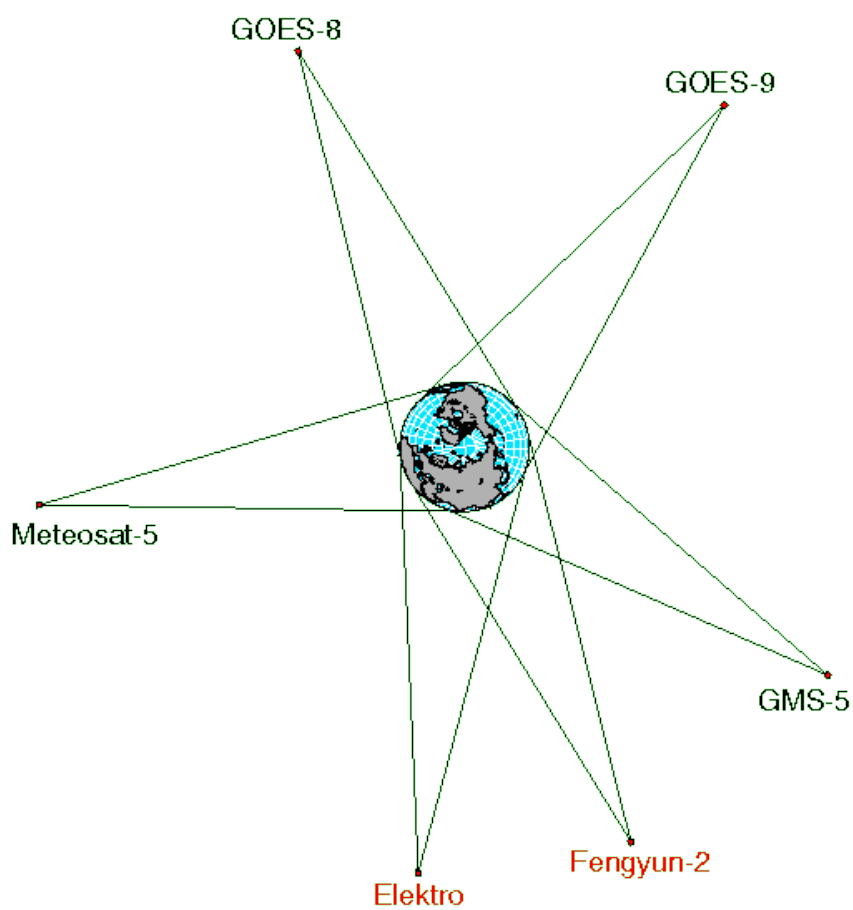


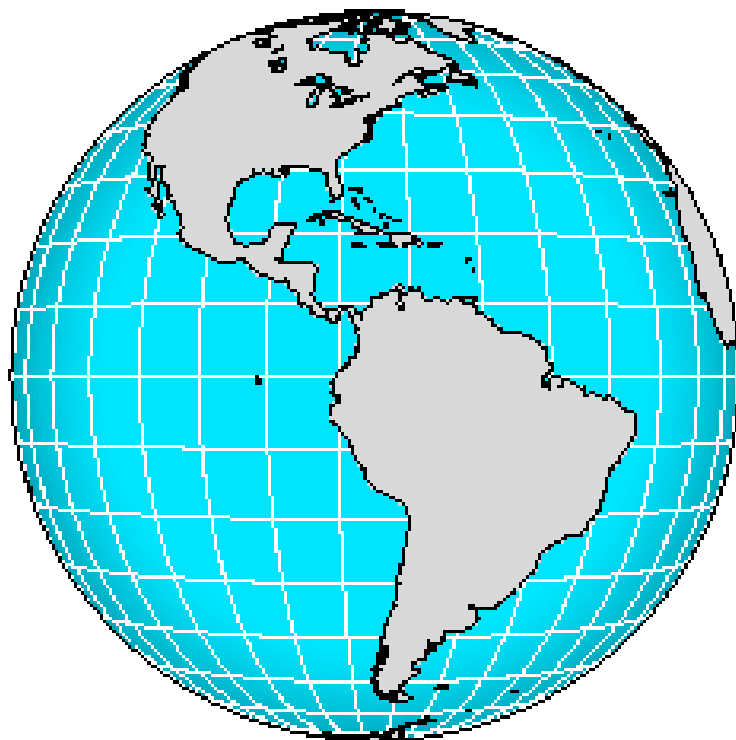
42% of the Earth surface



Longitude 0°

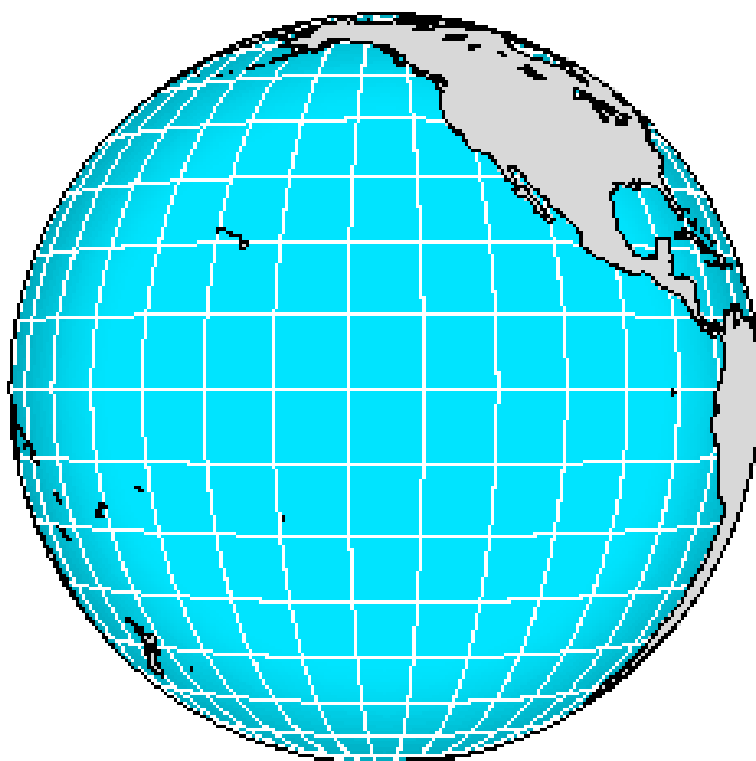
Meteosat-5





Longitude 75° W

GOES-8

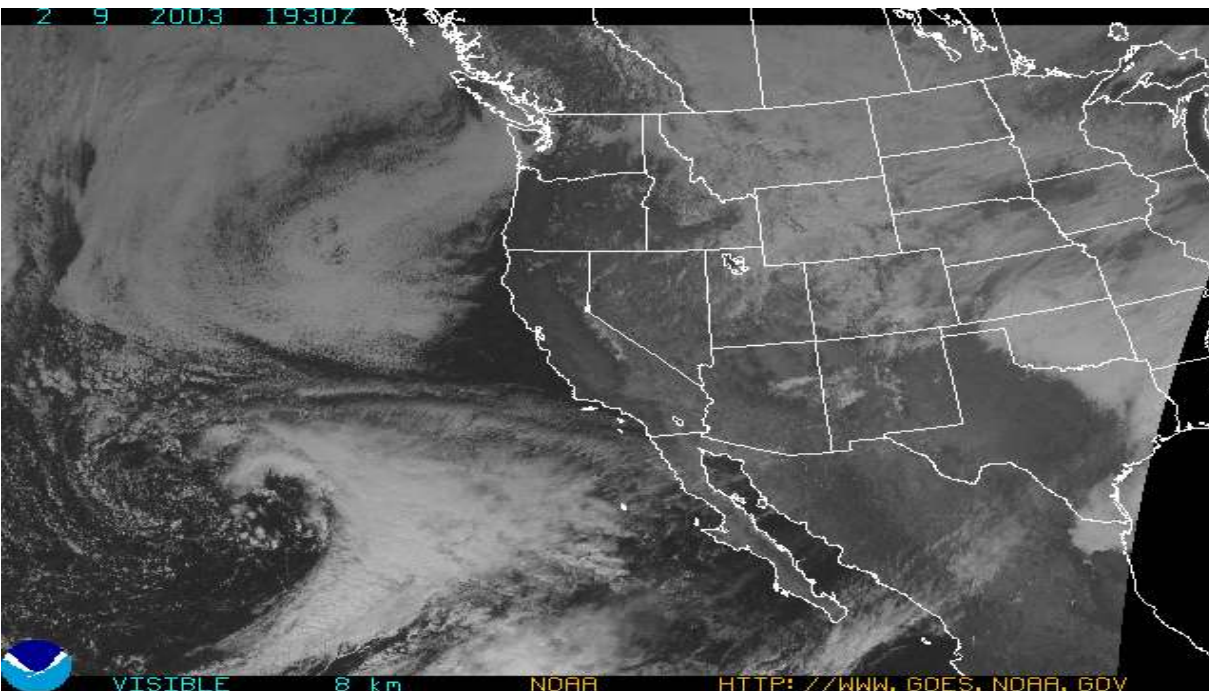


Longitude 135° W

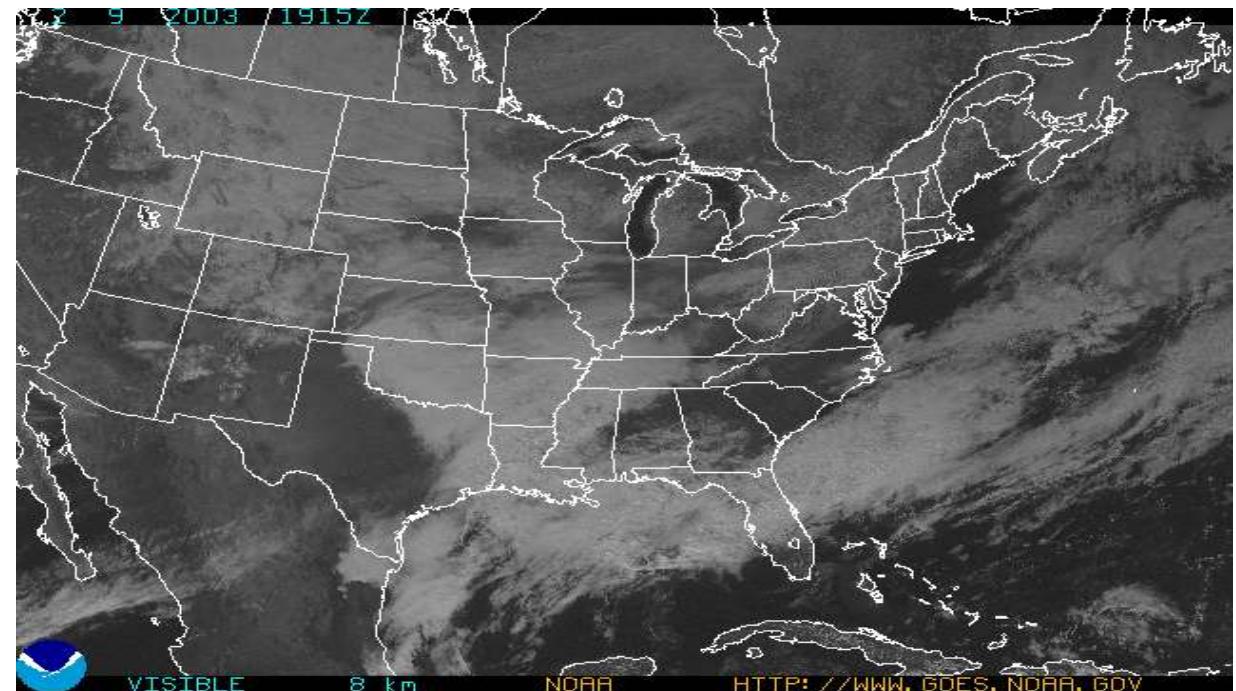
GOES-9



NOAA GOES Weather Satellites



GOES 10 – Western U.S.



GOES 8 – Eastern U.S.

GOES – Geostationary Operational Environmental Satellites

Images from February 9, 2003; <http://www.goes.noaa.gov/>



Advantages of the Geostationary orbit

- Continuous monitoring of the same area at any time
- Monitoring of great portion of the Earth surface
- Constant communication with the ground station allows receiving two subsequent images of the Earth in a short time lapse

Low Earth Orbits

LEO ($400 < h < 1000$ km)

$T \sim 100'$ (~ 14 orbits per day)

Quasi-polar Orbits ($i \sim 90^\circ$)

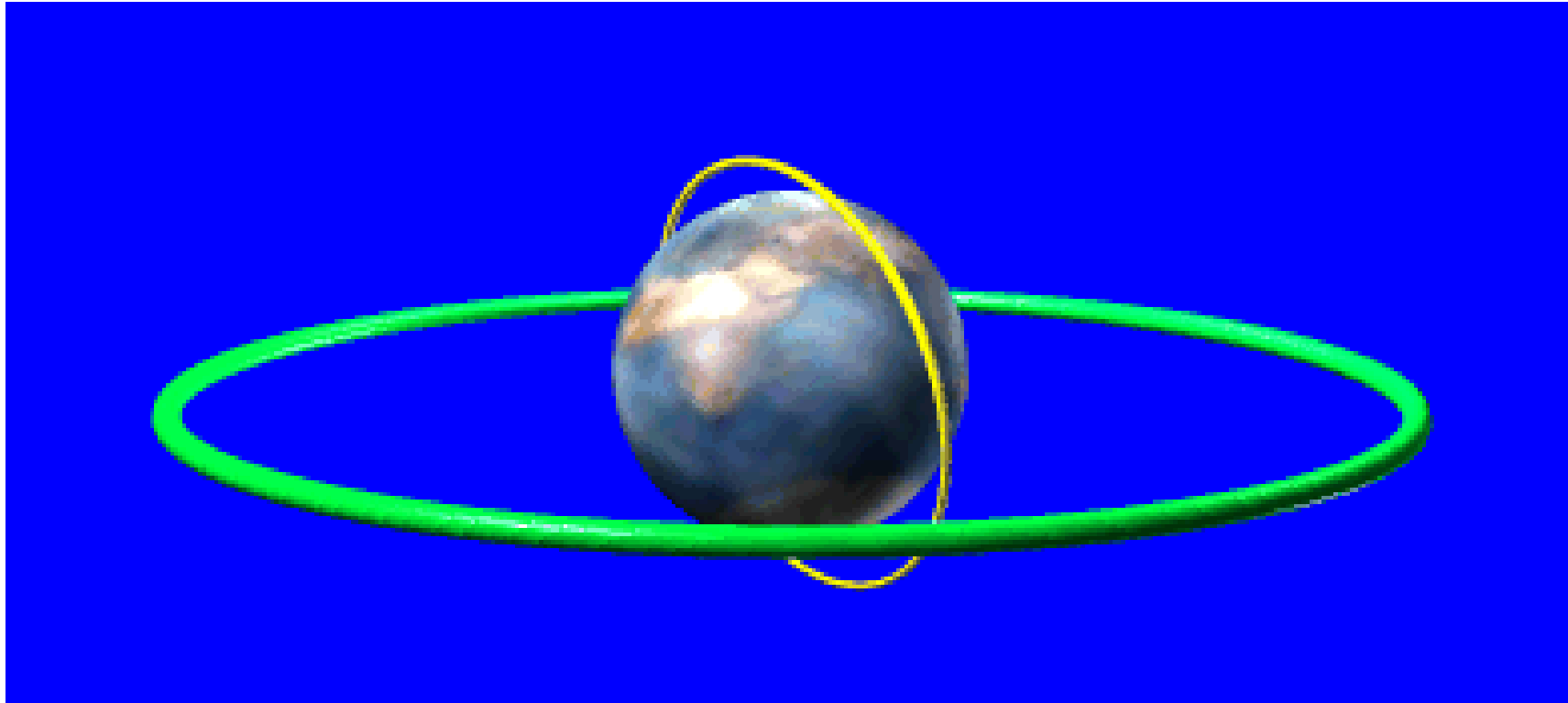
- Detailed view (high resolution)
- Almost total daily coverage of the Earth
- Poles observation

But

- Data can be received only when the satellite is in view of the ground station



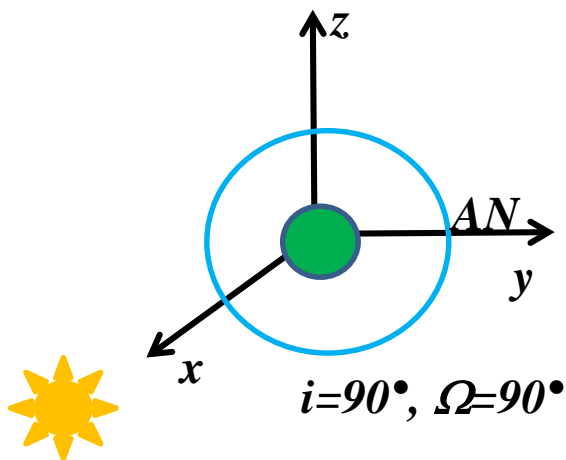
GEO and LEO Orbits in scale



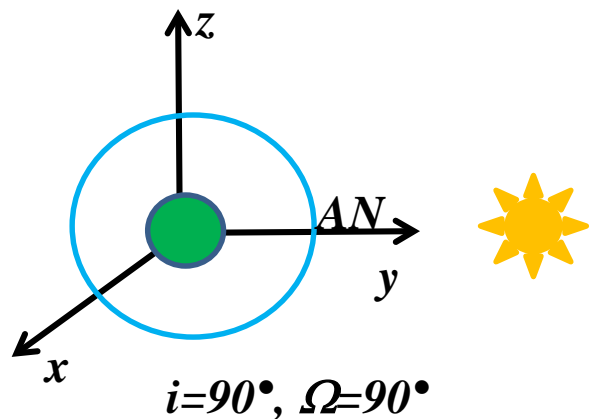
According to Kepler's laws,
the orbit of a satellite is fixed
in the space.

However,
its orientation wrt Sun changes
during the year.

Let's consider, for example,
a dawn-dusk orbit on March 21st



On June 21st, it is
a noon-midnight
orbit instead



The angle described by
the Sun – the Earth – the AN
is not constant during the year

As a consequence, the area observed by the satellite is lightened up by the Sun in a different way each day.

For those remote sensing measurements that require the Sun, the interpretation of data is made more difficult, if the illumination conditions change with time:

are different measurements due to changes in the illumination or to changes in the observed object?

For this reason, orbits with satellite passing over a region of interest at the same local time all over the year are preferred in remote sensing.

Therefore, the orbital plane must follow the Sun in its rotation “around” the Earth.

If the orbital plane rotates by the same angle described by the Sun during its apparent movement, then the satellite would pass over Zenith always at the same local time.

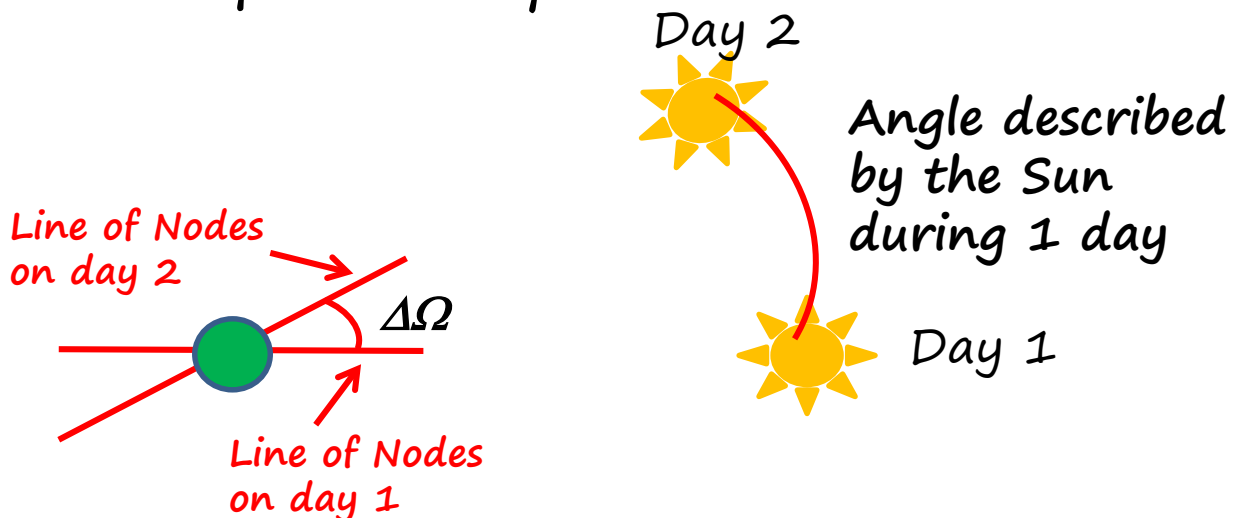
A rotation of the orbital plane is obtained modifying its Right Ascension of the Ascending Node Ω angle.

If after one day, the variation

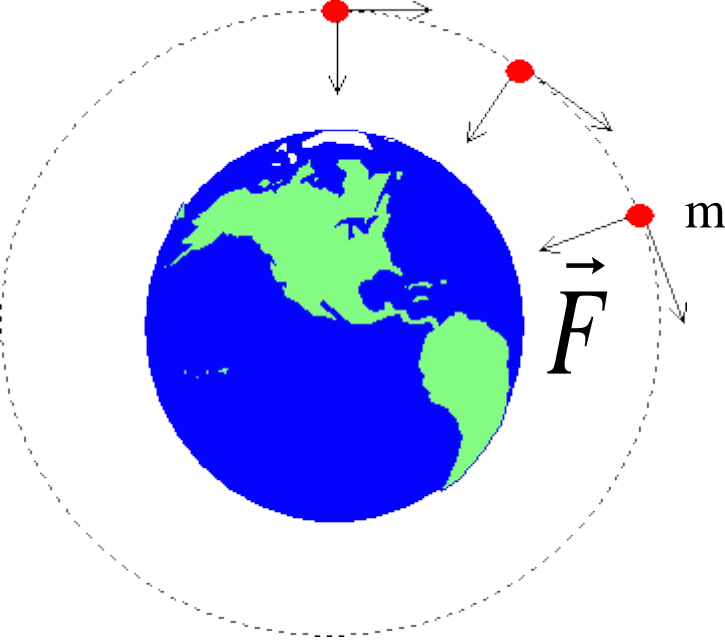
$$\Delta\Omega = 0.985674^\circ \sim 1^\circ$$

then the orbit is sun-synchronous

In the equatorial plane:



i.e., 90° every 3 months



$$\vec{F} = m\vec{a}$$

$$-\frac{GMm}{r^3} \vec{r} = m\ddot{\vec{r}} \quad (*)$$

Actually, we need to solve
a two-body problem

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

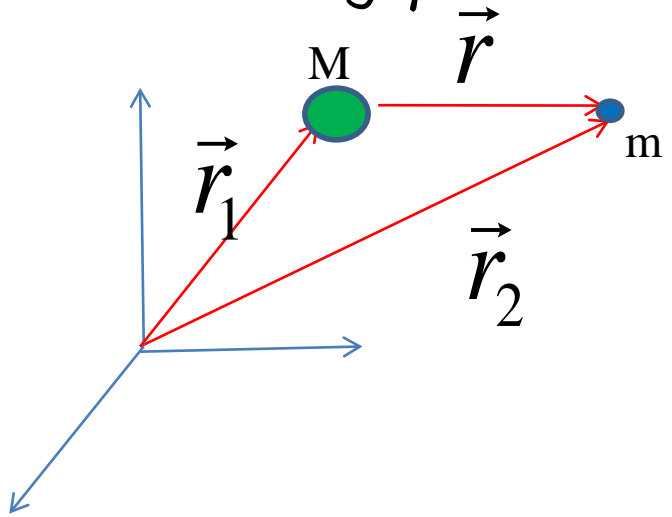
$$M\ddot{\vec{r}}_1 = \frac{GMm}{r^3} \vec{r} + \vec{f}_T$$

$$m\ddot{\vec{r}}_2 = -\frac{GMm}{r^3} \vec{r} + \vec{f}_S$$

$$\ddot{\vec{r}}_2 - \ddot{\vec{r}}_1 = -\frac{G(m+M)}{r^3} \vec{r} + \frac{\vec{f}_S}{m} - \frac{\vec{f}_T}{M}$$

$$\ddot{\vec{r}} = -\frac{G(m+M)}{r^3} \vec{r} + \vec{a}_e$$

If $\vec{a}_e = 0$ and $m \ll M$ we get
the Keplerian orbits *



Orbital perturbations

- Atmospheric drag
- Lunisolar (and other planets') attraction
- Earth oblateness
 - Solar radiation (pressure)
 - Solar wind (charged particles flux)

The Earth gravitational field is not uniform because the Earth is not a sphere, but more similar to an oblate spheroid.

Earth is flattened at the poles:

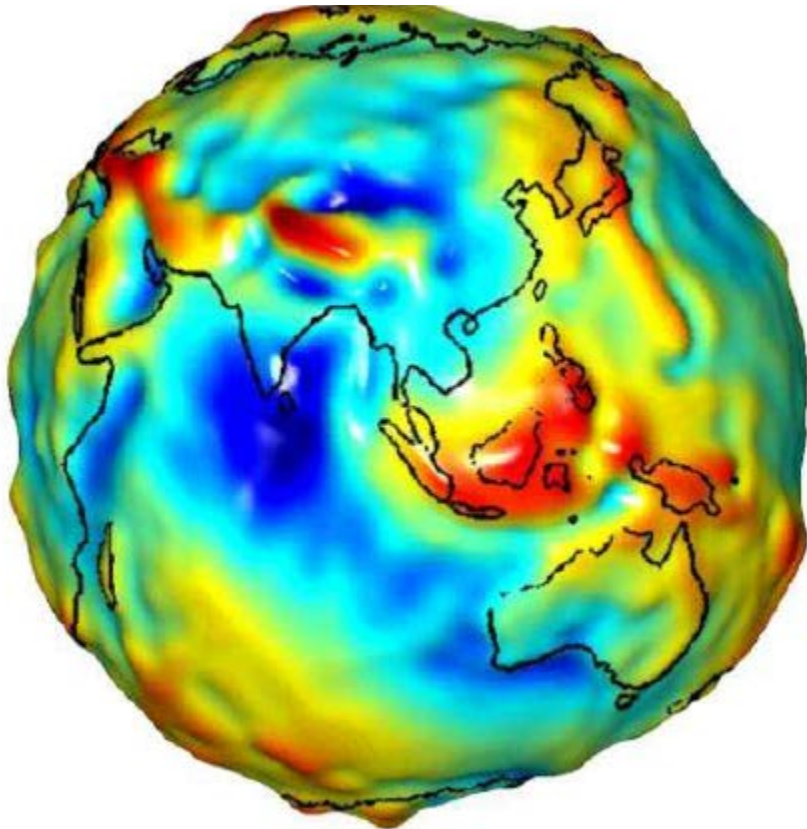
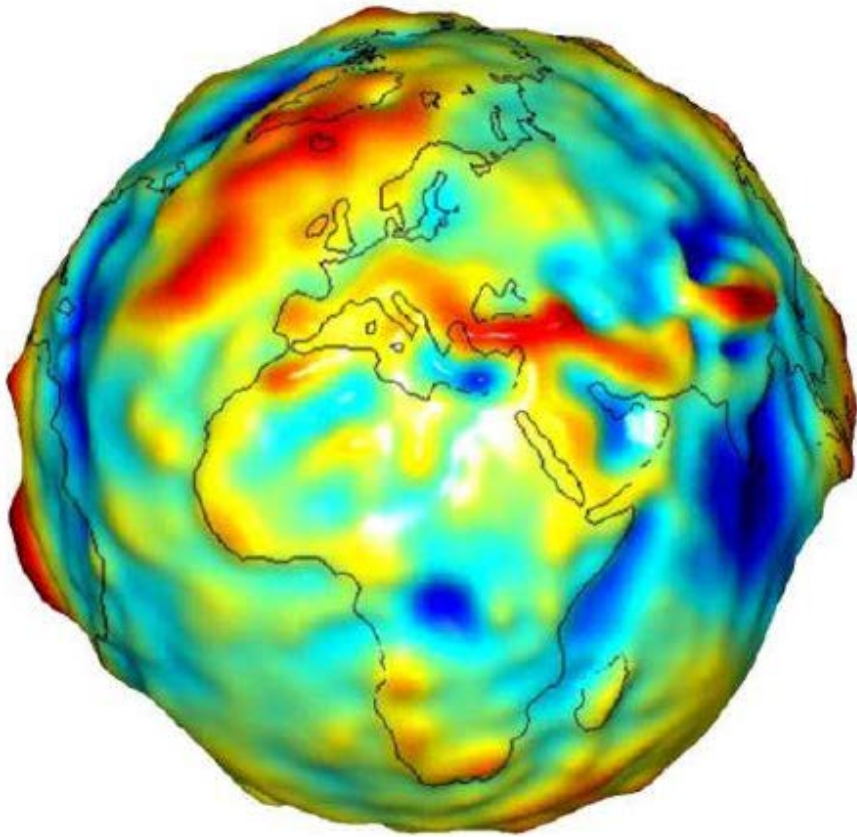
the difference between the equatorial diameter and the axial diameter is = 21.8 km

Furthermore, the equatorial section is not circular (equatorial bulge)

$$6378.2 \text{ km} \leq R_T \leq 6378.1 \text{ km}$$

Density variations affects the gravitational field

The geoid: the true physical shape of the Earth



Orbital plane precession

The line of nodes rotates on the equatorial plane around the z-axis

$$\frac{d\Omega}{dt} = -\frac{3}{2} \bar{\dot{\vartheta}} J_2 \left[\frac{R_T}{a(1-e^2)} \right]^2 \cos i$$

$$J_2 = \text{II zonal harmonic} = 1.08264 \cdot 10^{-3}$$
$$\bar{\dot{\vartheta}} = \text{mean angular velocity} = \sqrt{\frac{\mu}{a^3}}$$

The variation rate of the Ω angle depends on the satellite altitude and on the orbit inclination



For geostationary satellites

$$\frac{d\Omega}{dt} = -0.0134^\circ T^{-1}$$

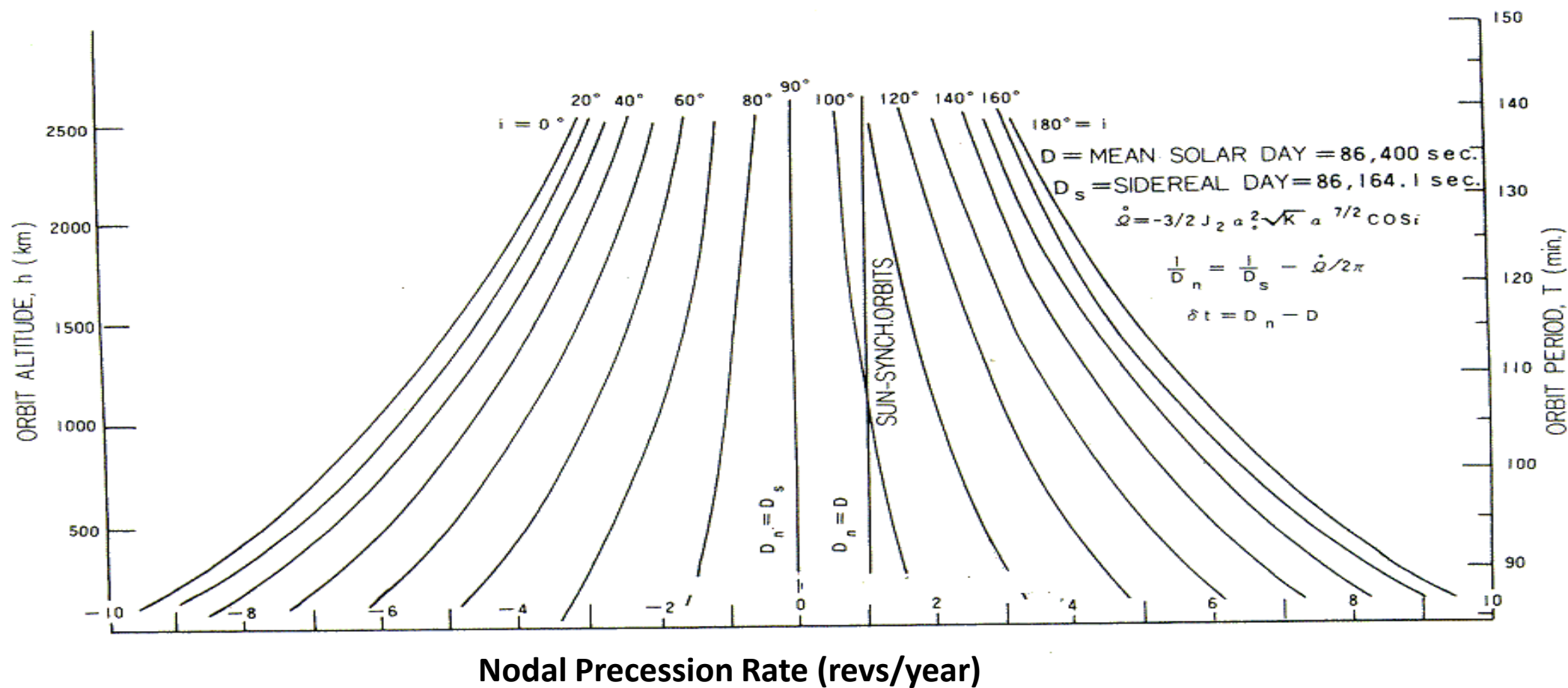


Figure 5.6.2 Satellite orbit precession and related viewing parameters, nodal precession rate Ω , orbit period T and orbit altitude h

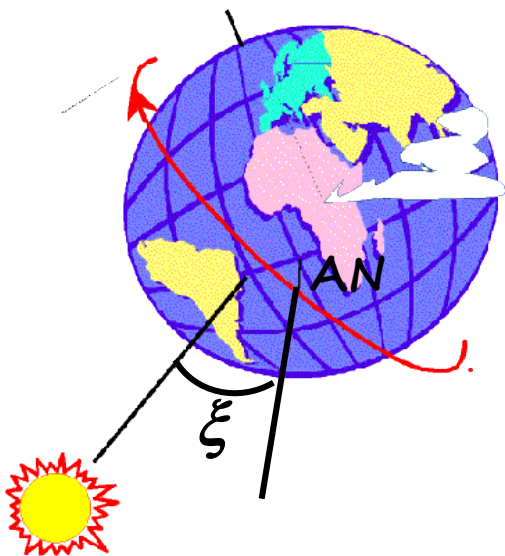
The orbital plane rotates naturally because of the Earth oblateness that produces an inhomogeneous gravitational field.

As a consequence, the right ascension of the ascending node precesses, i.e., Ω changes with time.

The Ascending Node precession is exploited in order to create sun-synchronous orbit

At a fixed satellite altitude, there is only one inclination angle that guarantees orbit sun-synchronism.

In LEO orbits $97^\circ < i < 99^\circ$



In sun-synchronous orbits, the angle ξ Sun-Earth-AN is constant, so that the satellite crosses the equatorial plane always at the same local time

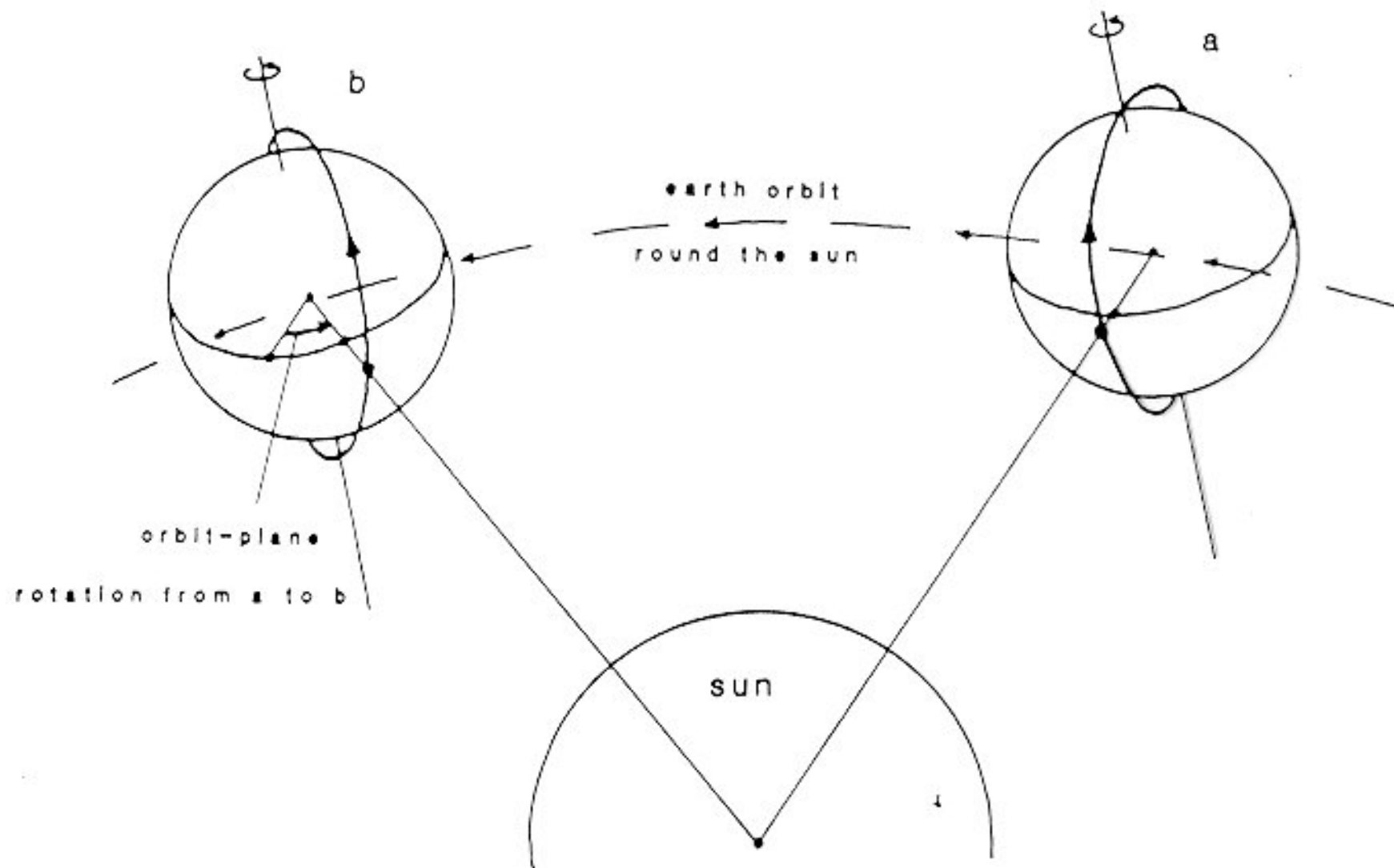
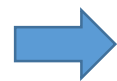
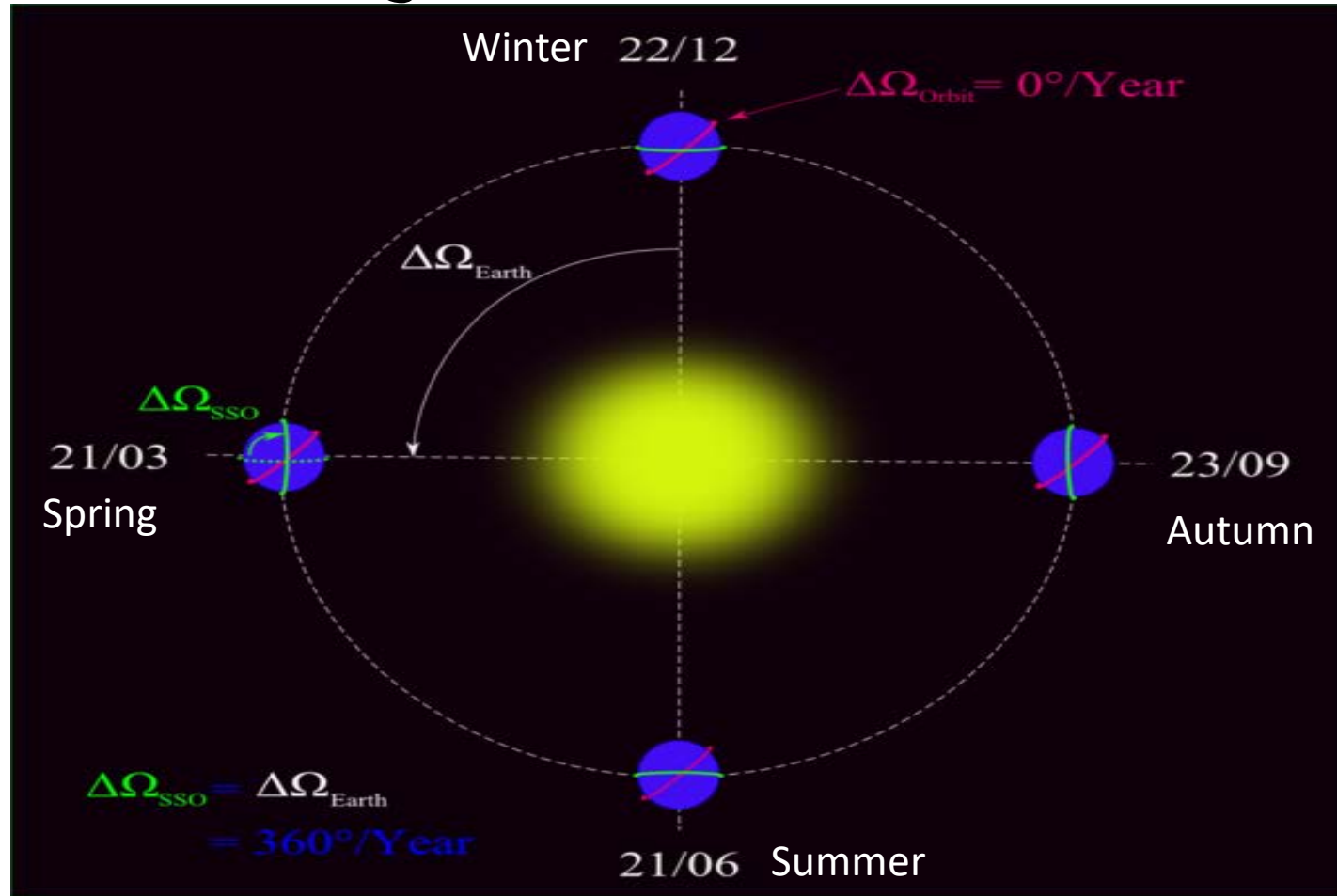


Fig. 2.5 – Orbit precession to achieve a sun-synchronous orbit



Sunsynchronous Orbit



Green: sunsynchronous orbit of a satellite. The orbit is a dawn-dusk orbit.

Magenta: Keplerian orbit that is fixed in space.



In sun-synchronous orbits,
instead of RAAN,
the local time of the satellite overpass
at the Ascending Node (or at the DN)
is given, i.e., the

Equator Crossing Time (ECT)

In circular orbits

$ECT = \text{local time at AN (or DN)}$

$ECT + 12 = \text{local time at DN (or AN)}$

If the illuminating source is the Sun,
the grazing angle must be $> 15^\circ$.

To get the best Signal/Noise Ratio,
illumination must be maximized, i.e.,

$9 \leq ECT \leq 16$ local time

Argument of perigee precession

$$\frac{d\omega}{dt} = \frac{3}{4} \bar{\dot{J}}_2 \left[\frac{R_T}{a(1-e^2)} \right]^2 (5 \cos^2 i - 1)$$

$$\frac{d\omega}{dt} = 0$$

When $i = 63.4^\circ$
 $i = 116.6^\circ$

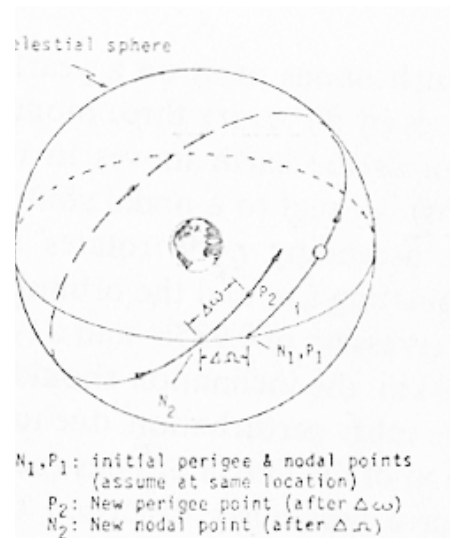
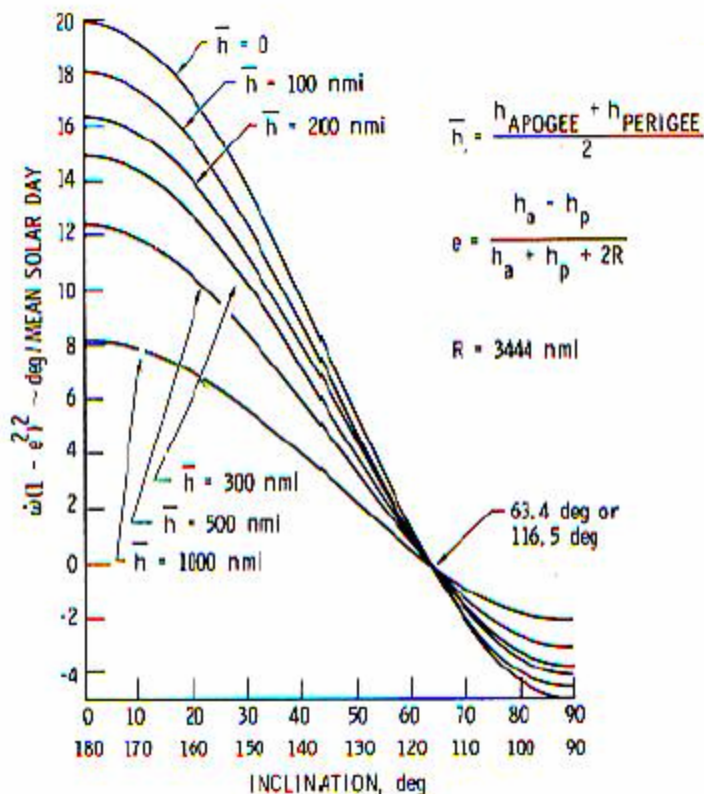
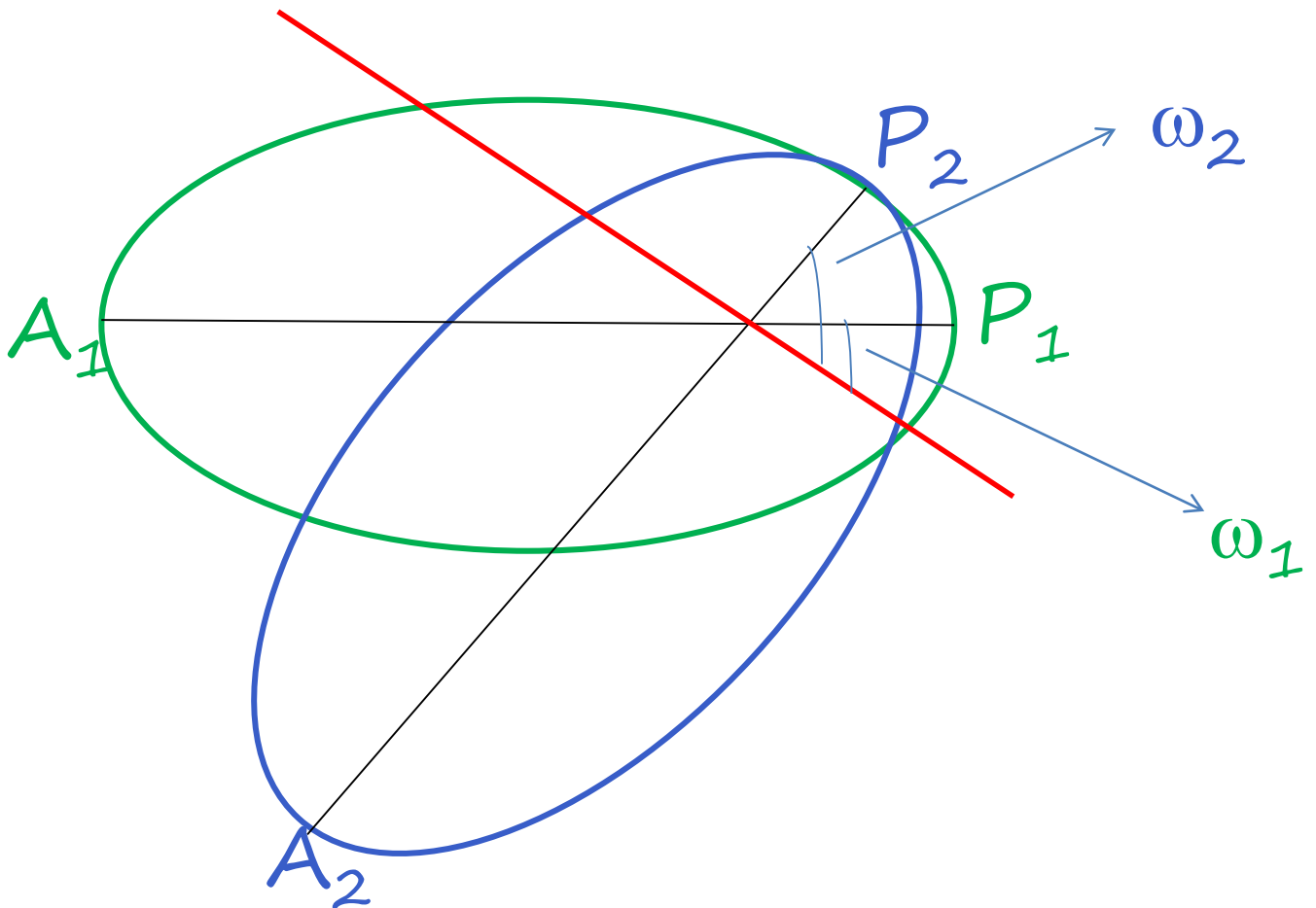


FIGURE 3-16. Apsidal rotation as a function of orbital inclination.

$$i=90^\circ$$

$$\Omega = \text{const}$$



At time t_1 :

P_1, ω_1

$t_2=t_1+dt$:

$P_2, \omega_2=\omega_1+d\omega$



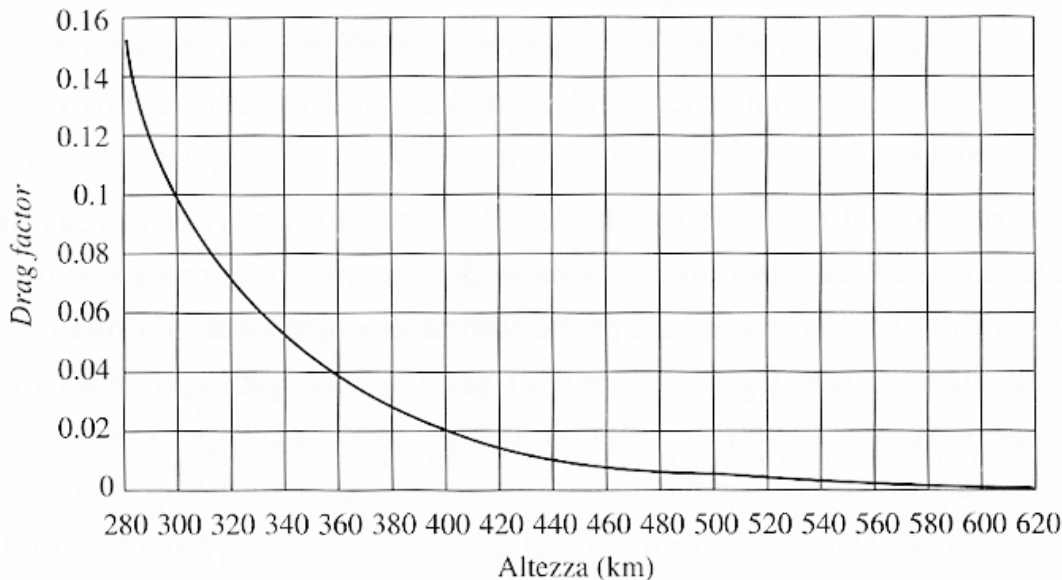


Fig. 5.8 - Andamento tipico del *drag factor* (adimensionale) in funzione dell'altezza del satellite.

The atmospheric drag effect rapidly grows with decreasing altitude (It is absent in geostationary orbits)

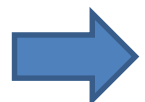
Atmosphere exerts a non-conservative force: alteration of the orbit major axis

It can be compensated by the on board propulsive system



The Sun “forces” the orbit center to rotate around the Sun:

- Variation of orbit inclination
- It is important in geostationary satellites.
- If not corrected, the orbit tilts $0.75 < i < 0.95^\circ$ in one year
- Correction is applied by means of the on board propulsive system: velocity increment $\sim 50\text{m/s}$ per year



Order of magnitude with respect to Central attraction of the existing perturbation forces

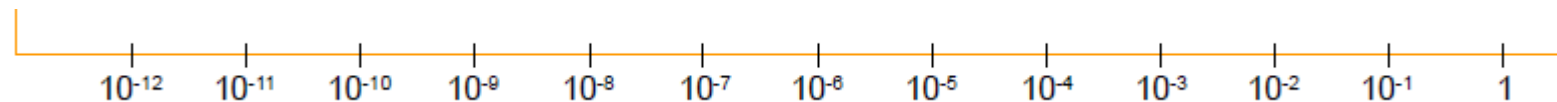
Solar radiation

Luni-solar attraction

Atmospheric drag ($150 < \text{height} < 1000 \text{ km}$)

Earth oblateness

Central attraction



Ground Track

Nadir Trace

It is the satellite orbit projected onto the Earth surface.

It is the ensemble of points (subsatellite points) that the satellite sees at its Nadir during its orbit.

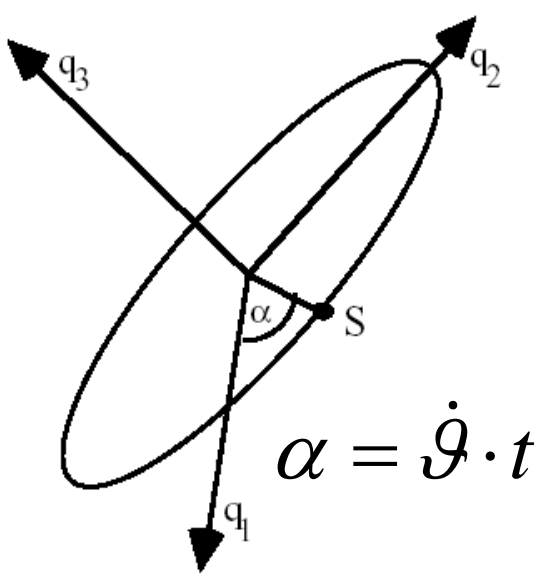
You need to combine the satellite motion along the orbit with the rotational motion of the Earth.

The satellite angular velocity on a circular orbit is

$$\dot{\theta} = \sqrt{\frac{\mu}{r^3}} \quad \text{with } r = R_T + h$$

The Earth angular velocity is

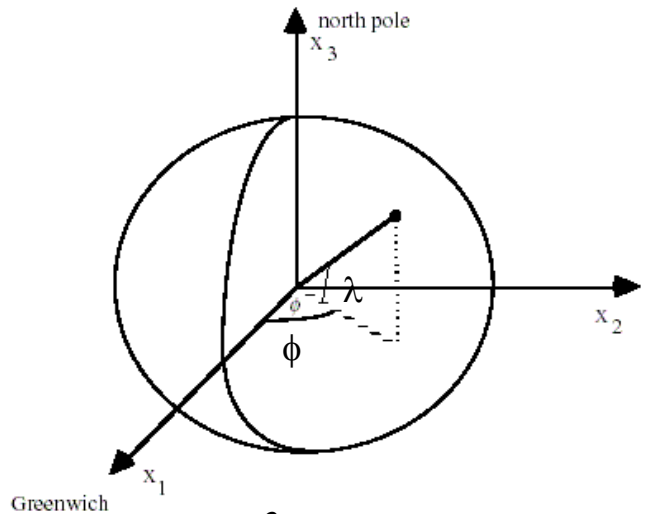
$$\dot{\theta}_T = \frac{2\pi}{D_s} = 72.92115 \cdot 10^{-6} \text{ rad/s}$$



- q_1 axis is directed from the Earth center to the Ascending Node
- q_2 axis is contained in the orbital plane
- q_3 is orthogonal to the orbital plane
- α is the anomaly, i.e., the angle described by the satellite along its motion on the orbital plane

In the reference frame where the Earth is non-rotating, each point on the Earth surface is described by the coordinates of latitude λ and longitude ϕ

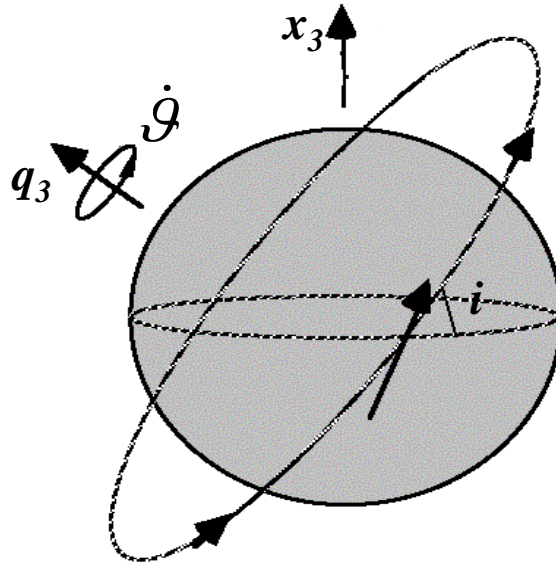
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos \lambda \cos \phi \\ \cos \lambda \sin \phi \\ \sin \lambda \end{pmatrix}$$



Assuming q_1 coincident with x_1 , to go from the satellite reference frame to the non-rotating Earth frame, you need to apply the transformation:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

i =inclination angle of the orbital plane



In order to describe the satellite motion with the coordinates of latitude and longitude, it is necessary to make the transformation:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos \lambda \cos \phi \\ \cos \lambda \sin \phi \\ \sin \lambda \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{pmatrix} \cos \alpha \\ \sin \alpha \\ 0 \end{pmatrix}$$

$$\lambda = \sin^{-1}(\sin i \sin \alpha)$$

$$\phi = \operatorname{tg}^{-1}(\cos i \cdot \operatorname{tg} \alpha)$$



If the Earth were non-rotating,
 λ and ϕ would replicate
 after one orbital period

But the Earth rotates with velocity $\dot{\vartheta}_T$
Additionally, the orbital plane precesses with
velocity $\dot{\Omega}$

Both motions produce a rotation around axis x_3 ,
and consequently they affect only longitude ϕ

$$\lambda = \sin^{-1}(\sin i \sin \alpha)$$

$$\phi = \operatorname{tg}^{-1}(\cos i \cdot \operatorname{tg} \alpha) - (\dot{\vartheta}_T - \dot{\Omega})t$$

Combining the satellite orbital motion with the
Earth rotation and the Ascending Node
precession, the nadir track moves westwards
after each orbital period.

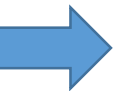
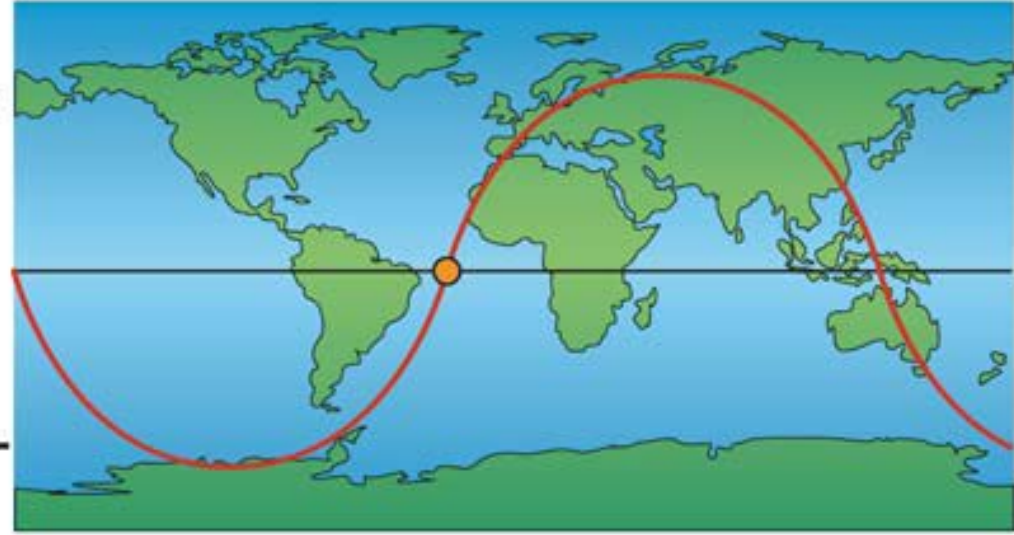
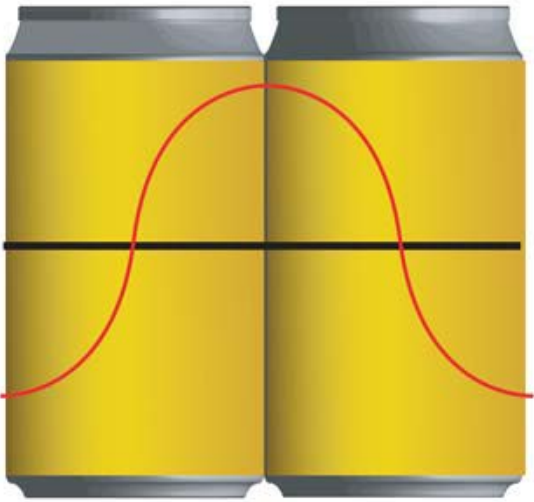
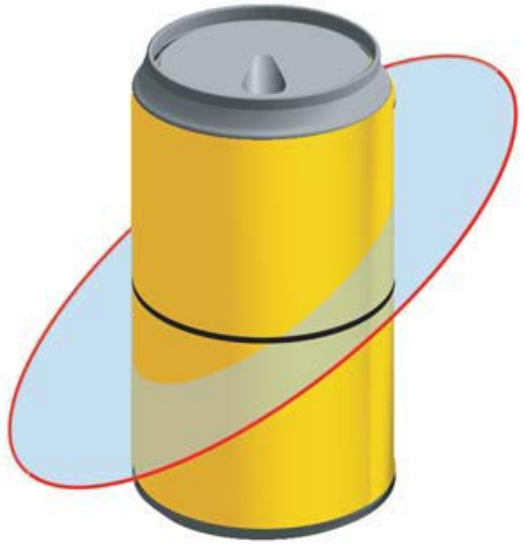


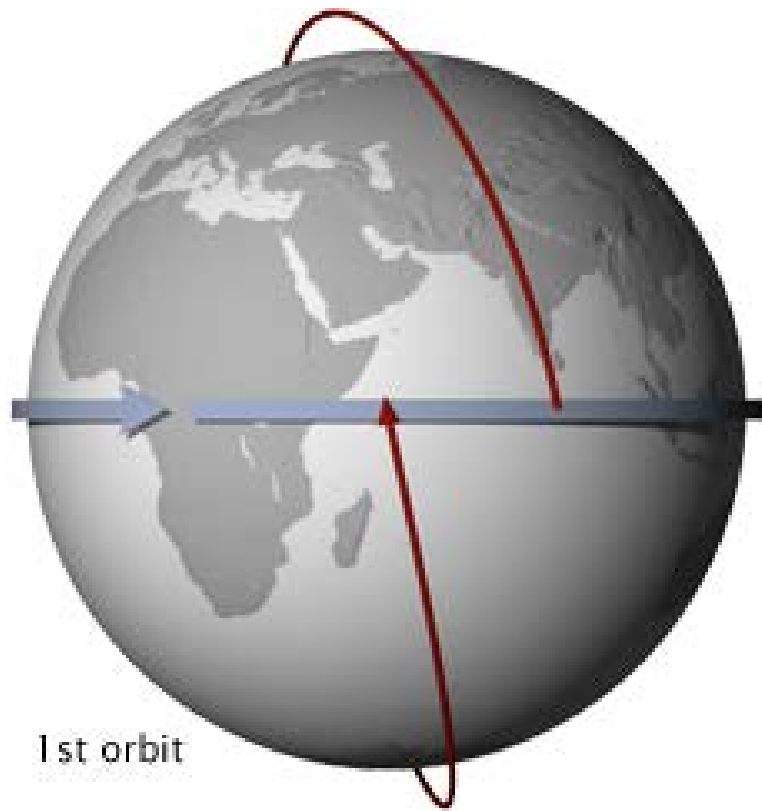
This displacement is measured by the
fundamental interval:

$$S = 360 \frac{\dot{\vartheta}_T - \dot{\Omega}}{\dot{\vartheta}} \quad \text{in degrees}$$

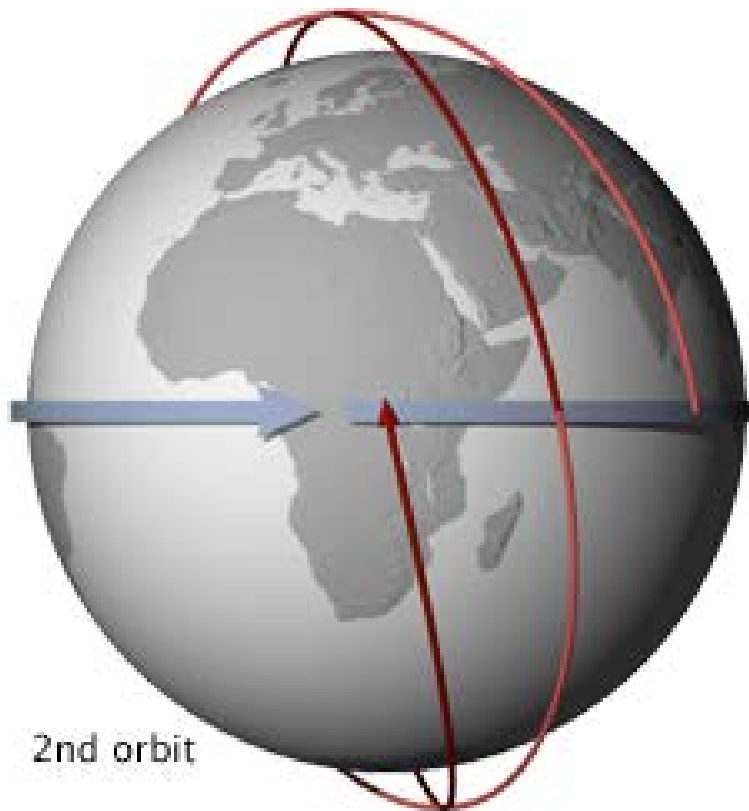
that provides the angular distance (in degrees)
between two subsequent crossings of the
equator.



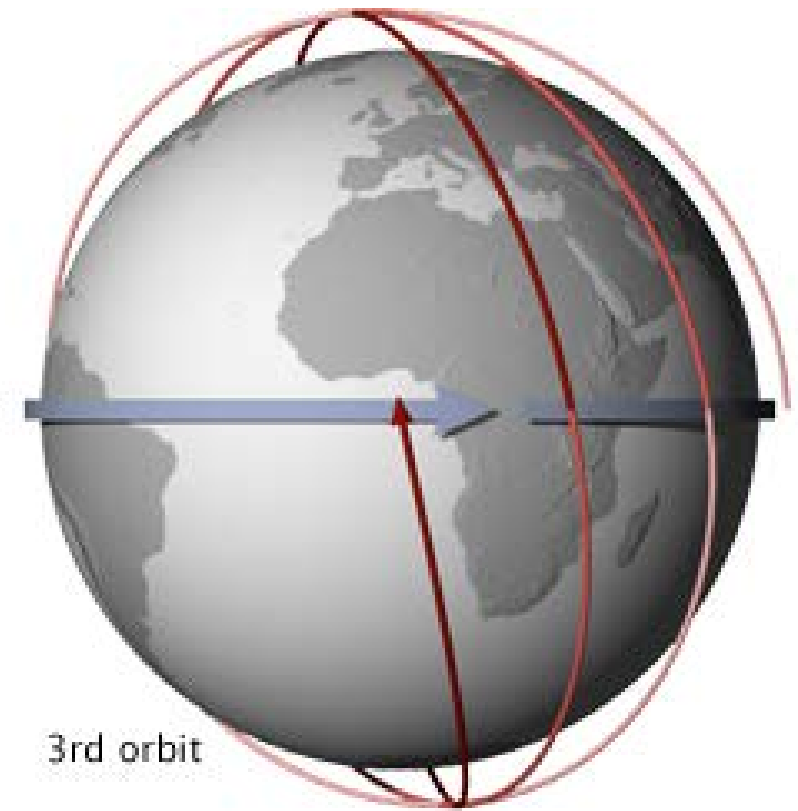




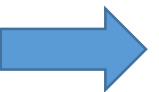
1st orbit

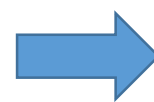
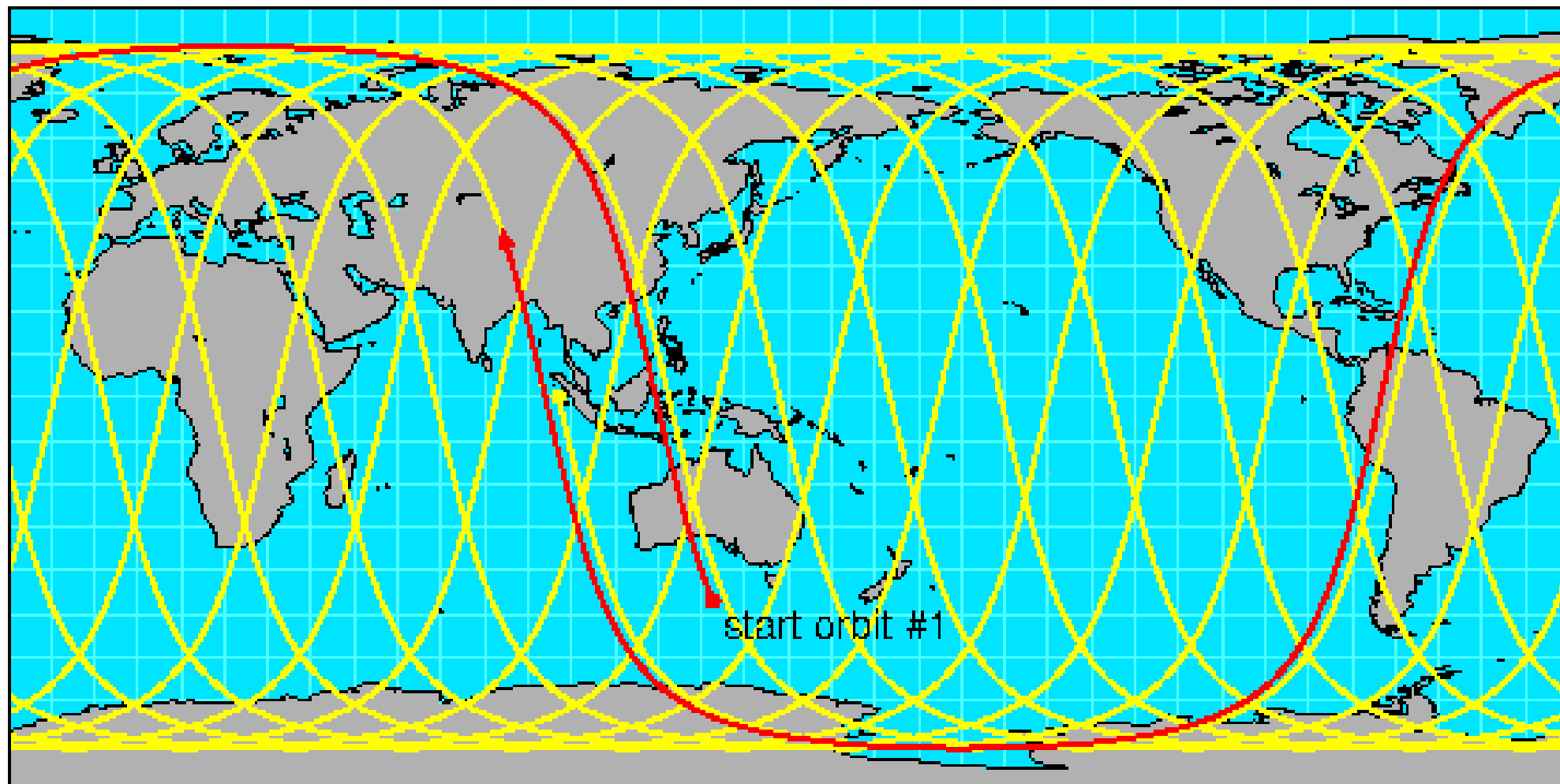


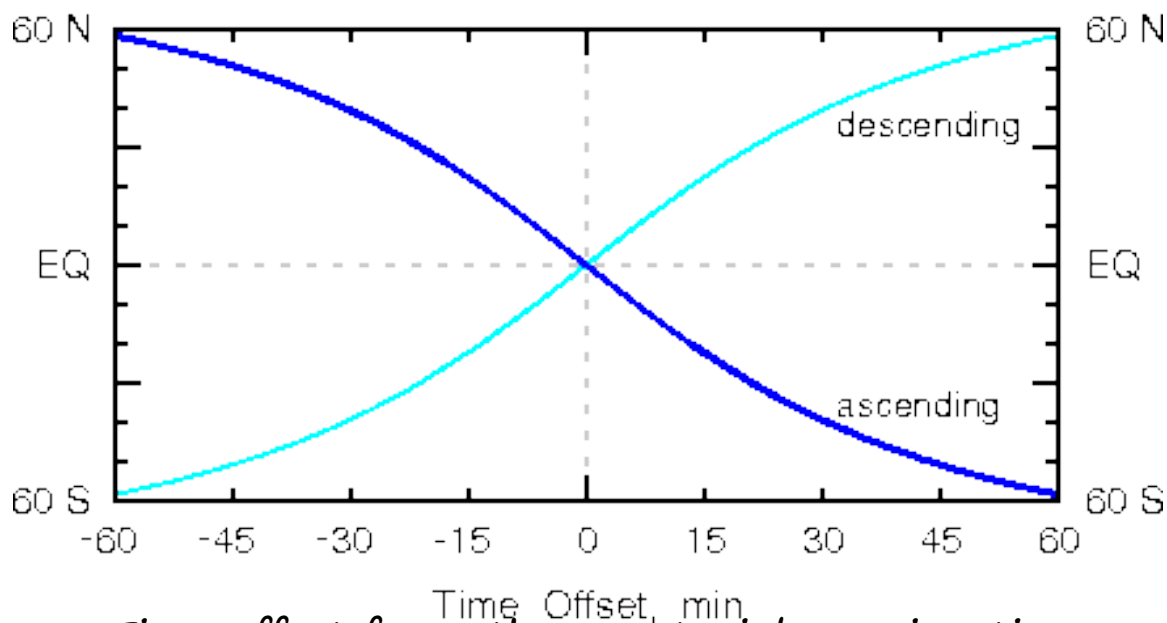
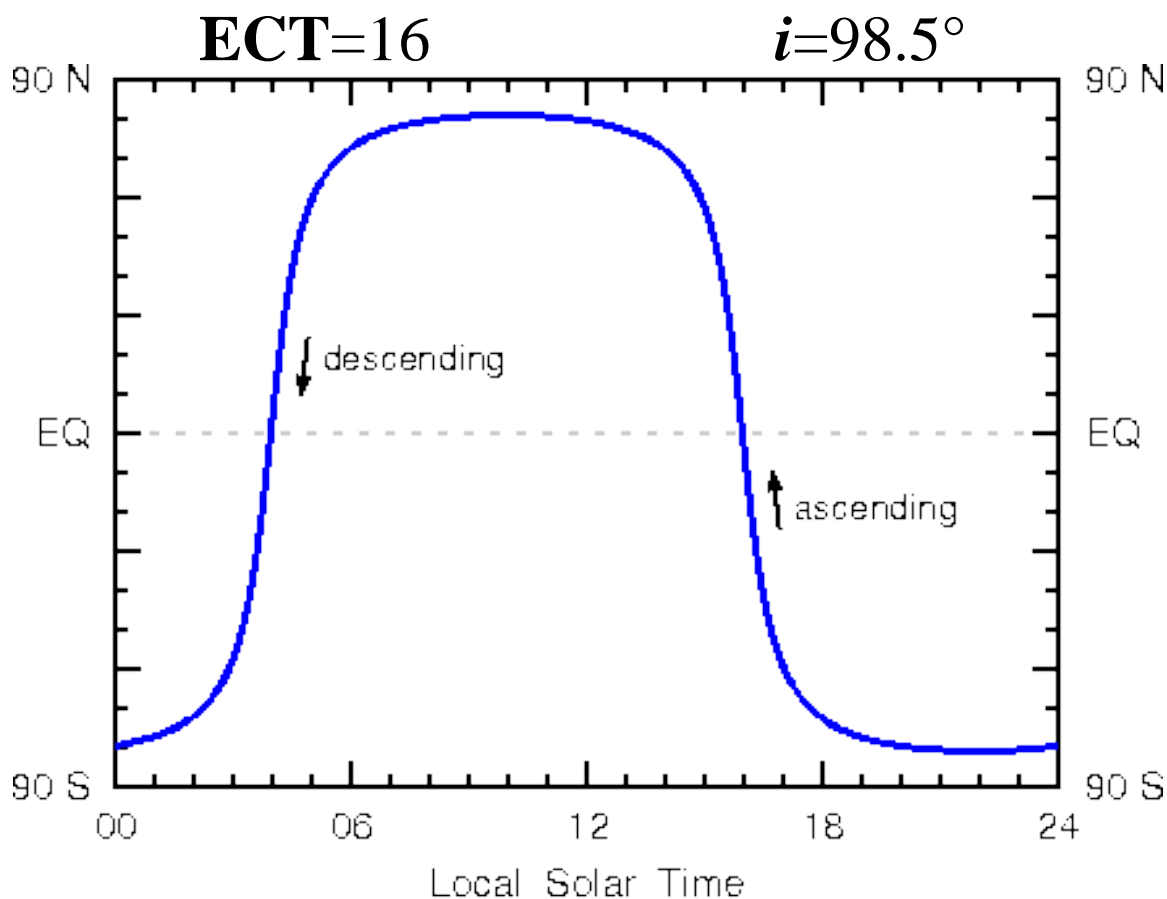
2nd orbit



3rd orbit







*Time offset from the equatorial crossing time
as a function of latitude*

*This bottom plot is applicable to every satellite with the same
inclination and orbital period (~100min)*

The Repeat Cycle is the interval of time after which a ground track exactly repeats

D =days in a repeat cycle

O =orbits in a repeat cycle

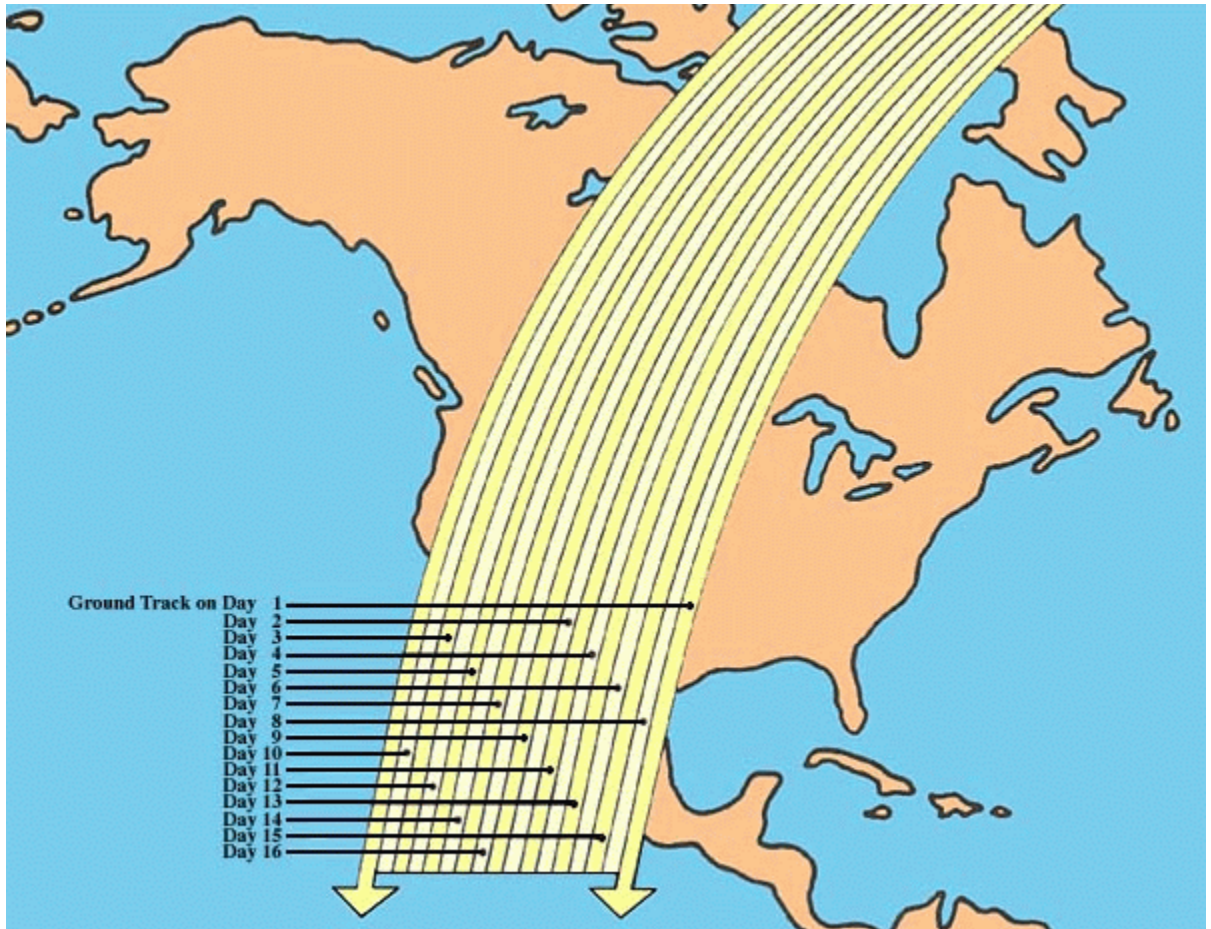
$Q=O/D$ =orbits per day

Then:

$$S=360 \cdot D/O=360/Q \text{ [deg]}$$

In general, D is not an integer number, except for sun-synchronous orbits.

In sun-synchronous orbits Q is a ratio of integer numbers.



The fundamental interval is crossed once a day for D days.

The sub-interval is defined as

$$S_i = S/D$$

i.e., the distance between two adjacent ground tracks.

(Landsat gives two adjacent ground tracks after 7 days)

S and S_i can be measured also in Km.

$$S \text{ [km]} = S \cdot R_T \cdot \cos \lambda$$

$$S_i \text{ [km]} = S_i \cdot R_T \cdot \cos \lambda$$

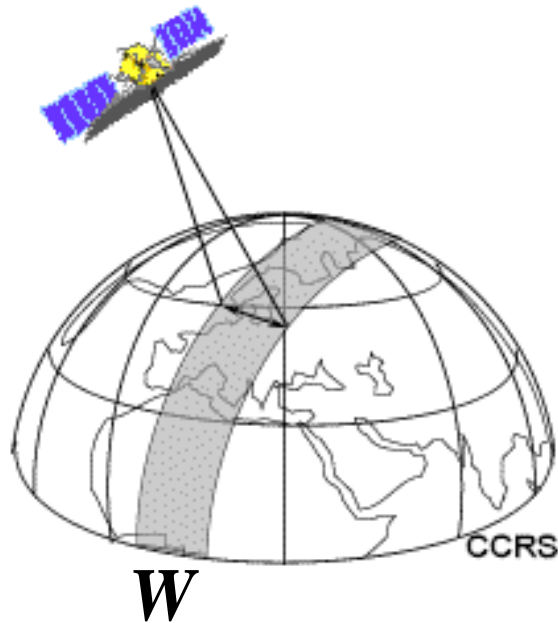
For Landsat

$$S = 2752 \text{ km}$$

$$S_i = 172 \text{ km}$$

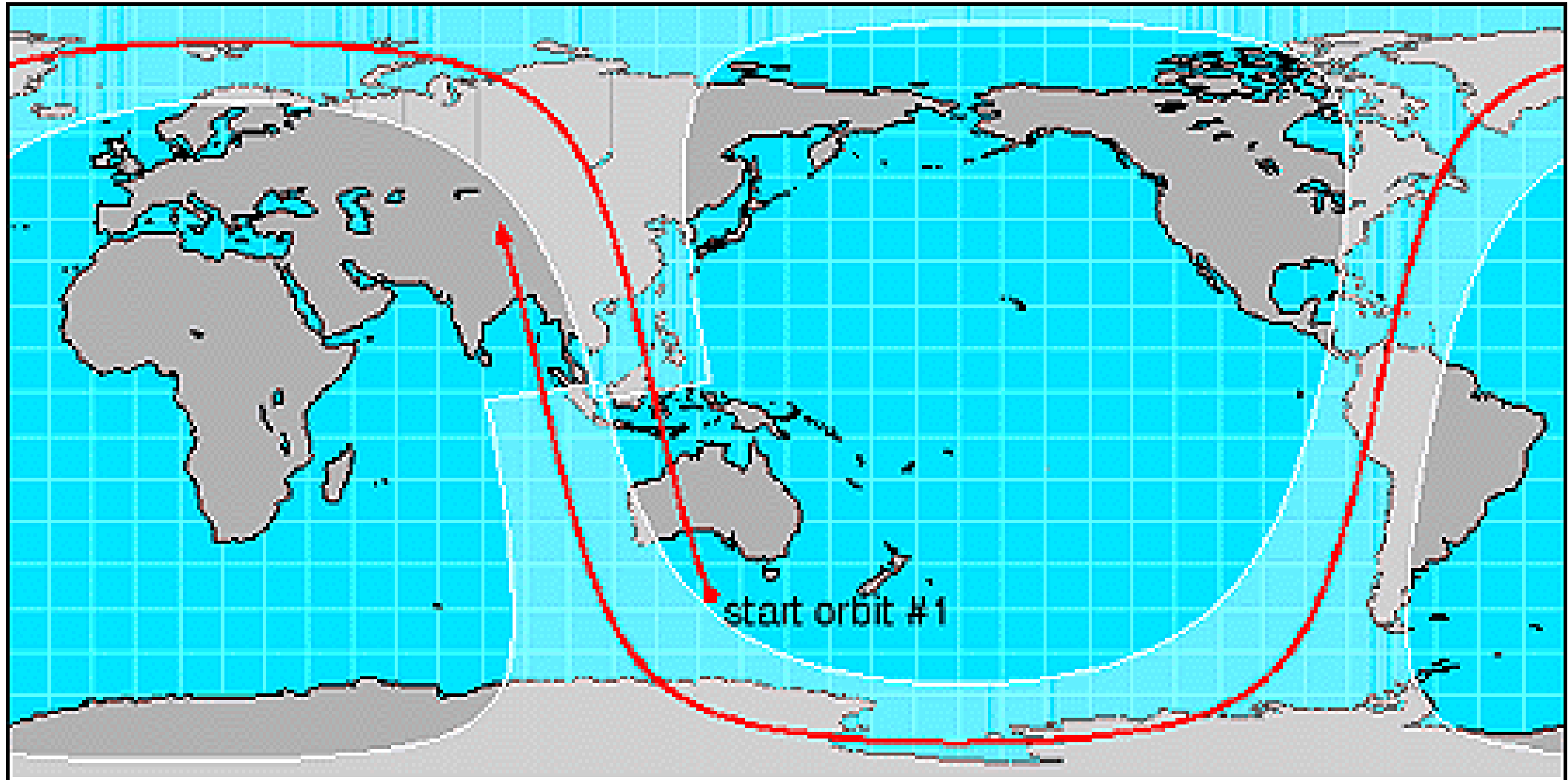
at the equator.

Coverage is obtained combining the ground track with the sensor swath W

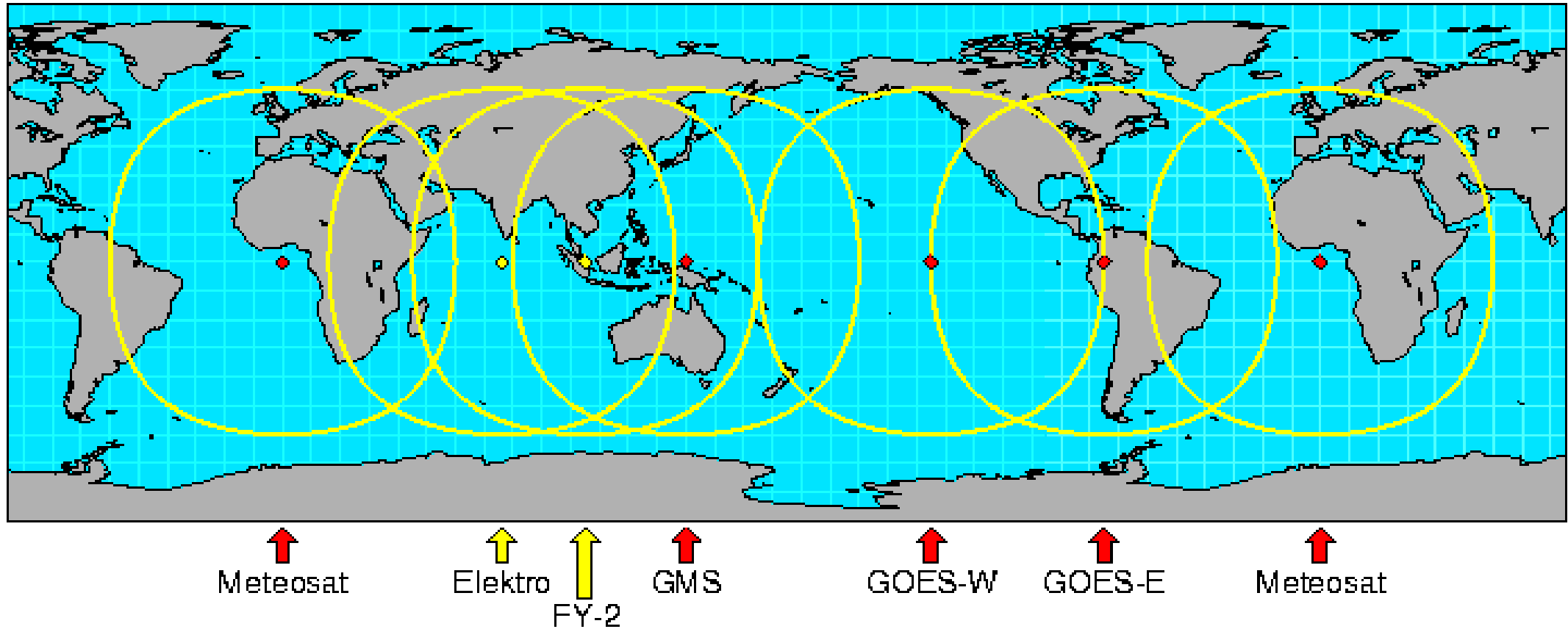


AVHRR on-board NOAA/POES satellites

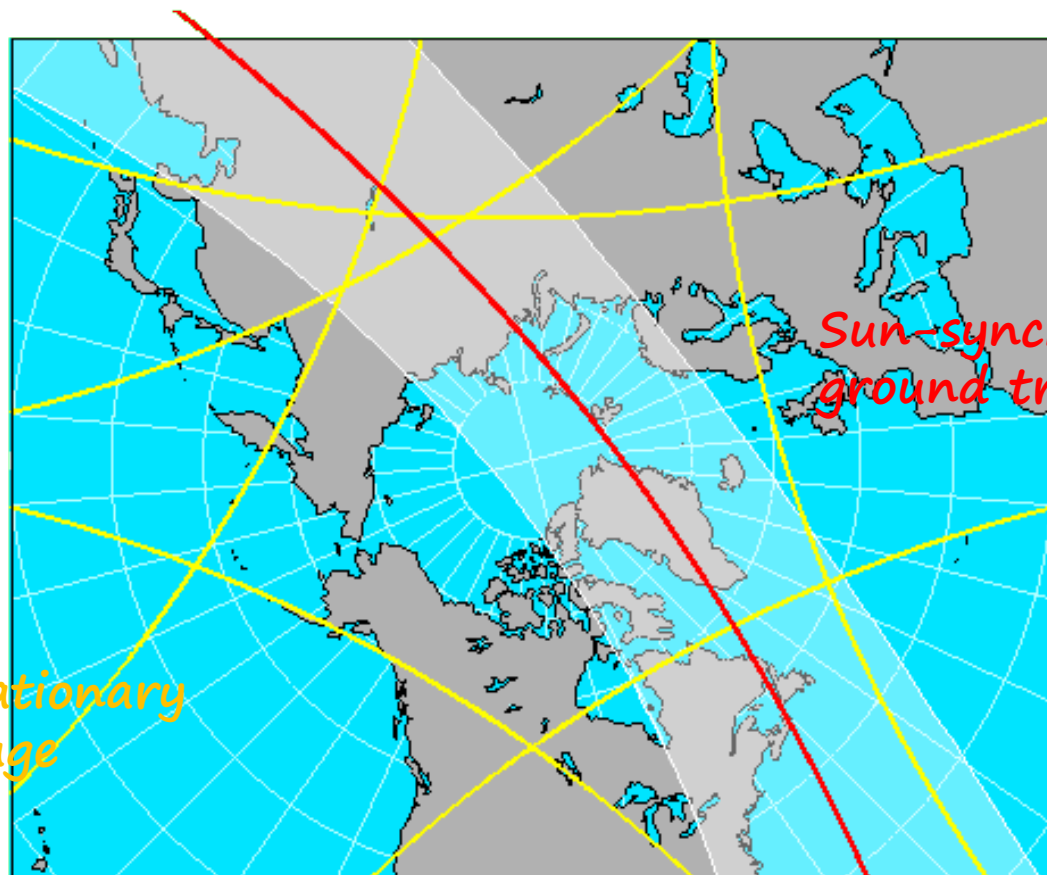
$h \sim 850 \text{ km}$ $i \sim 98.8^\circ$ $W \sim 3000 \text{ km}$ $S \sim 2800 \text{ km}$



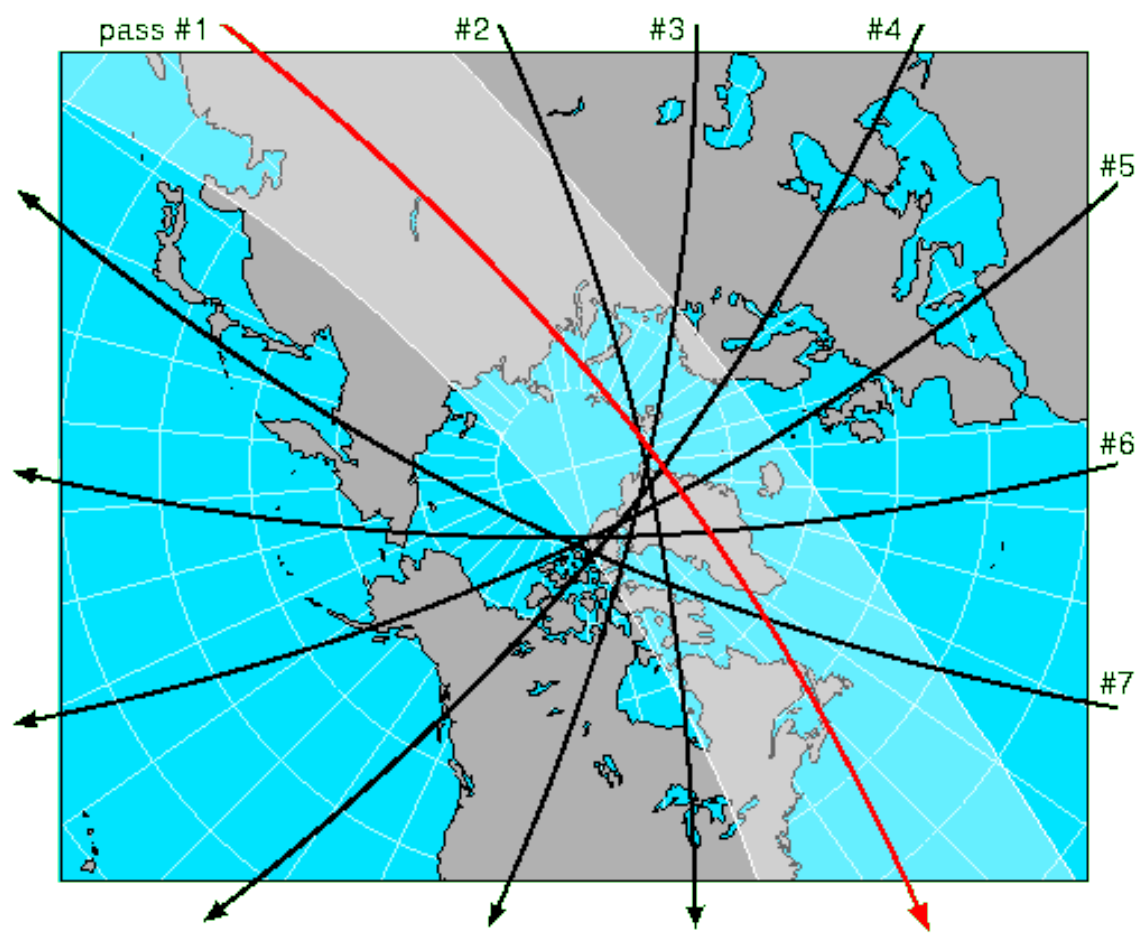
Global Geostationary Satellite Coverage



Geostationary
coverage



Sun-synchronous
ground track



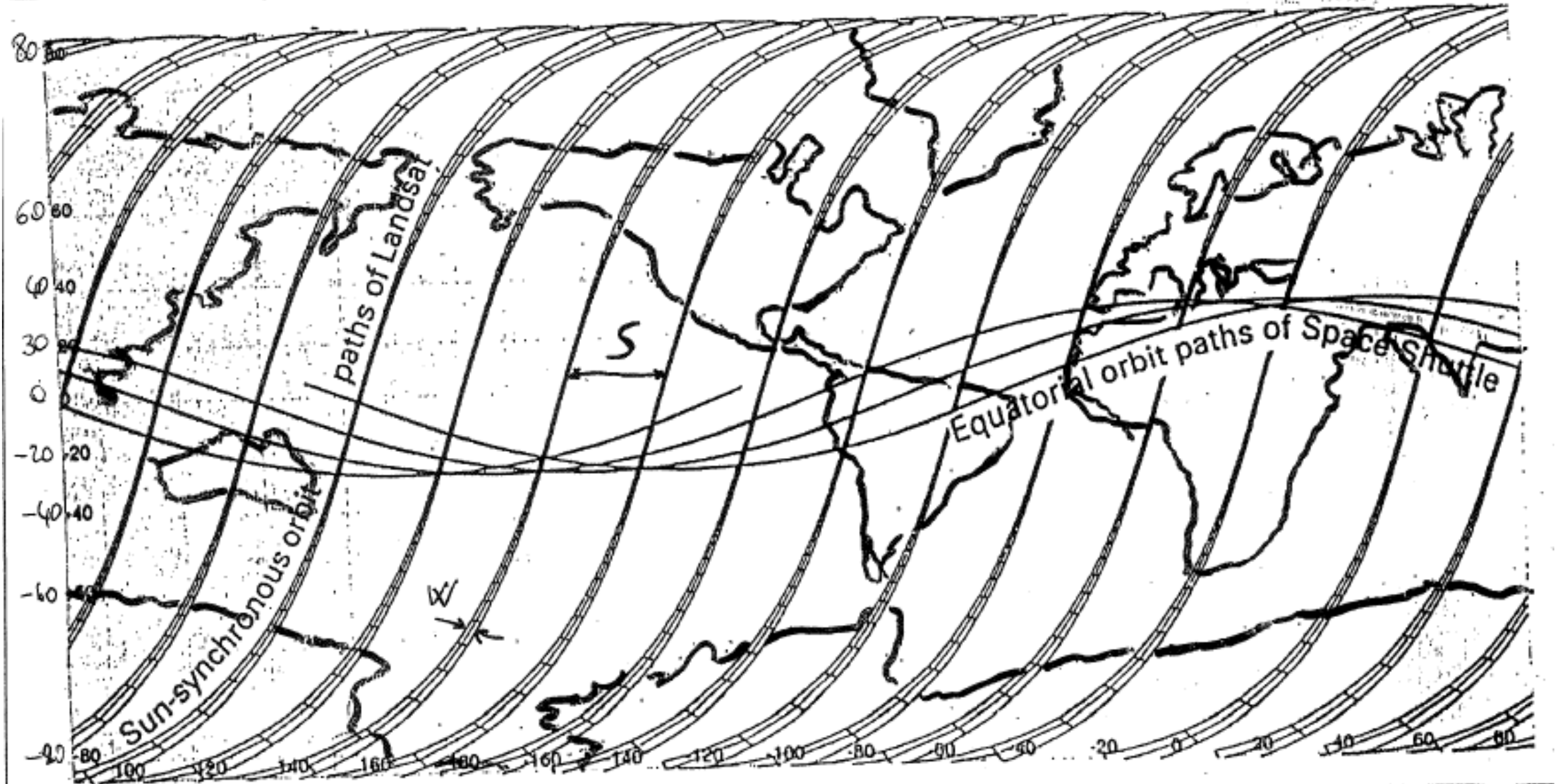
If $W/\sin i \geq S[\text{km}, \lambda=0]$
the complete Earth coverage
is reached daily

For sun-synchronous orbits,
to have complete daily coverage
 $W \geq S[\text{km}, \lambda=0]$


If $W \geq S_i$, Earth complete coverage
is reached in a Repeat Cycle

$S=2752 \text{ km}$

$W=185 \text{ km}$



Revisit Time is the time lapse between two observations of the same site.

$$\text{Revisit Time} \leq \text{Repeat Cycle}$$

It depends on

- the orbit
- the sensor
- the site latitude

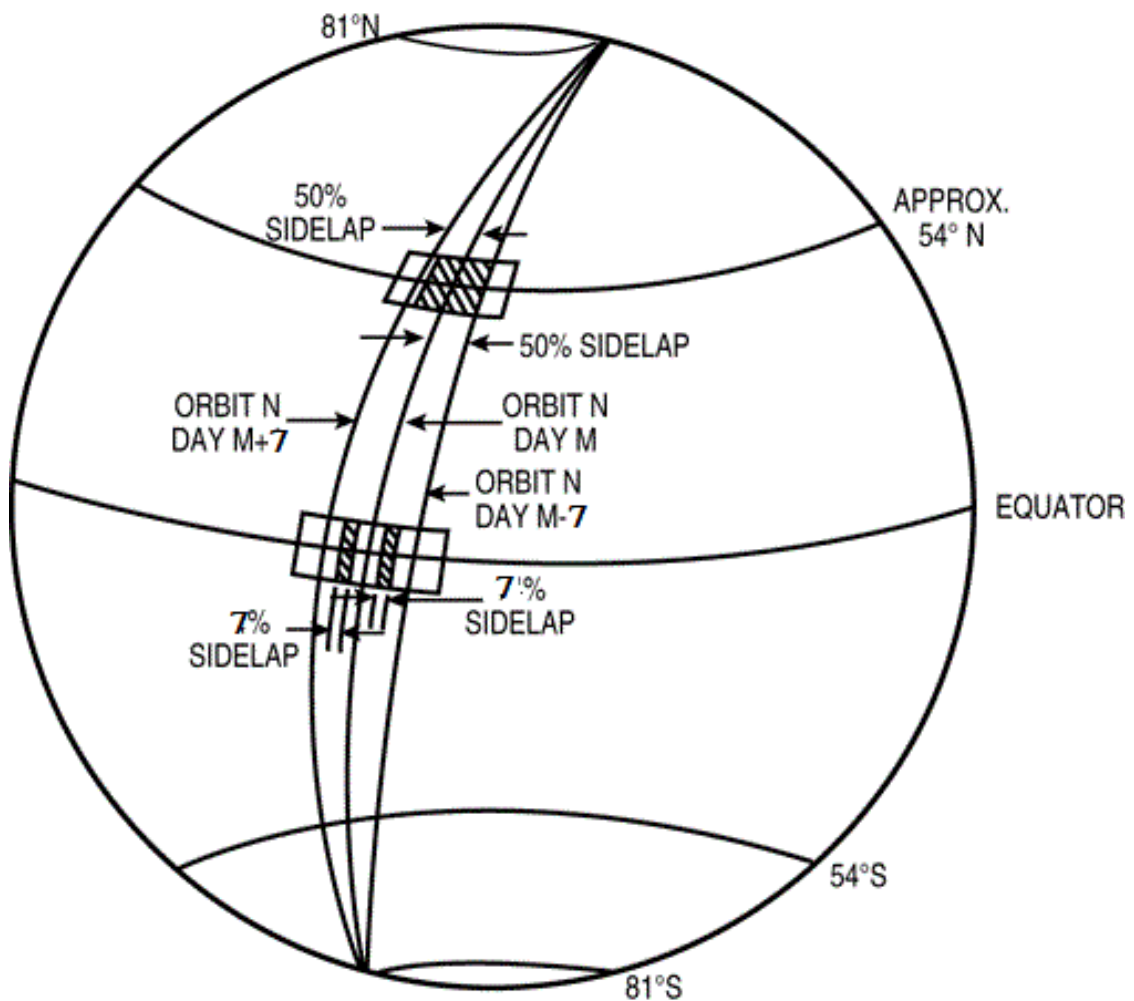
Polar areas have a very low
Revisit Time

Satellite constellations shorten the
Revisit Time

Landsat 4 + Landsat 5 provide a
Revisit Time ~ 8 days

Landsat Thematic Mapper (TM) has a swath
 $W=185$ km,
giving overlapping at Equator of 7.6%.

At latitudes $\sim 60^\circ$, side overlapping is $\sim 54\%$

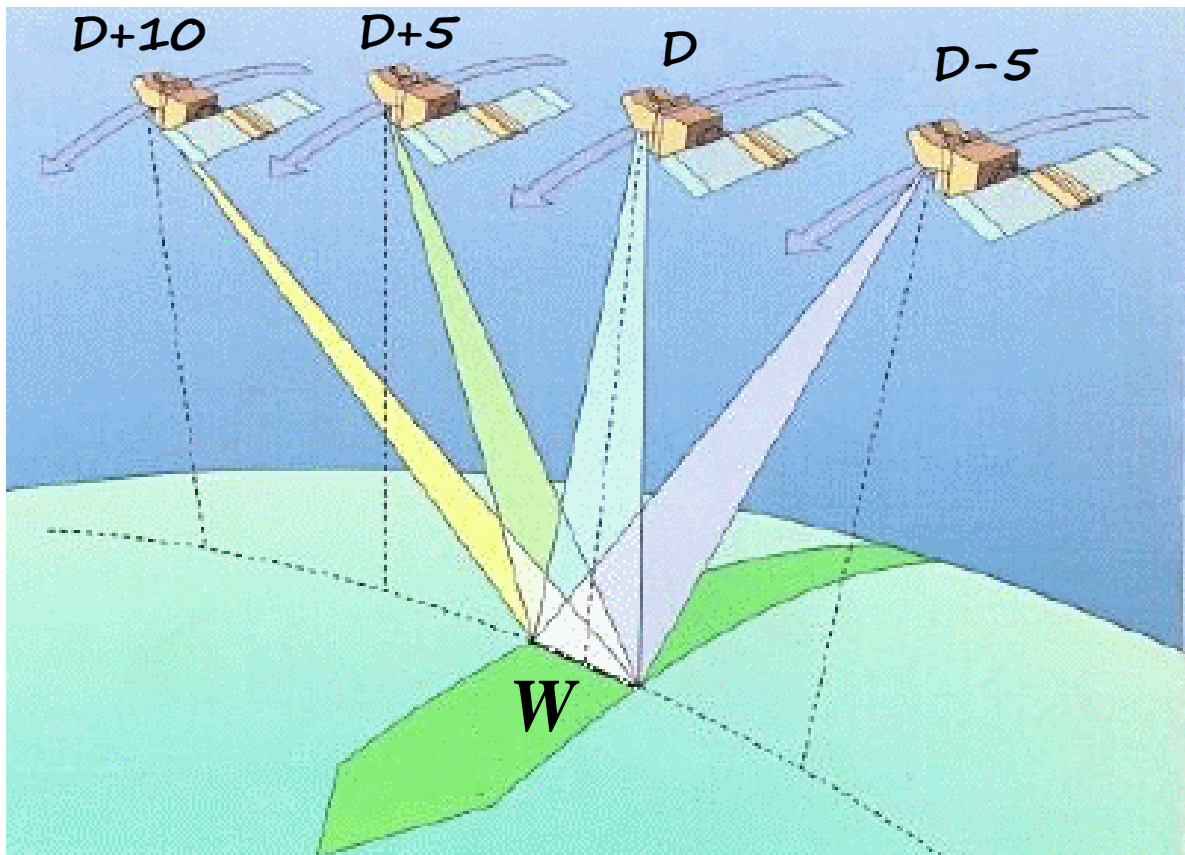


The french satellite SPOT is on a sunsynchronous orbit at ~ 830 km, with a Repeat Cycle = 26 Days

But SPOT sensor can acquire also off-nadir, so that

in a Repeat Cycle

- the sites at equator can be revisited 7 times
- the sites at latitudes $=45^\circ$ can be revisited 11 times



European Remote Sensing (ERS) Satellite

had a Repeat Cycle of 35 days.

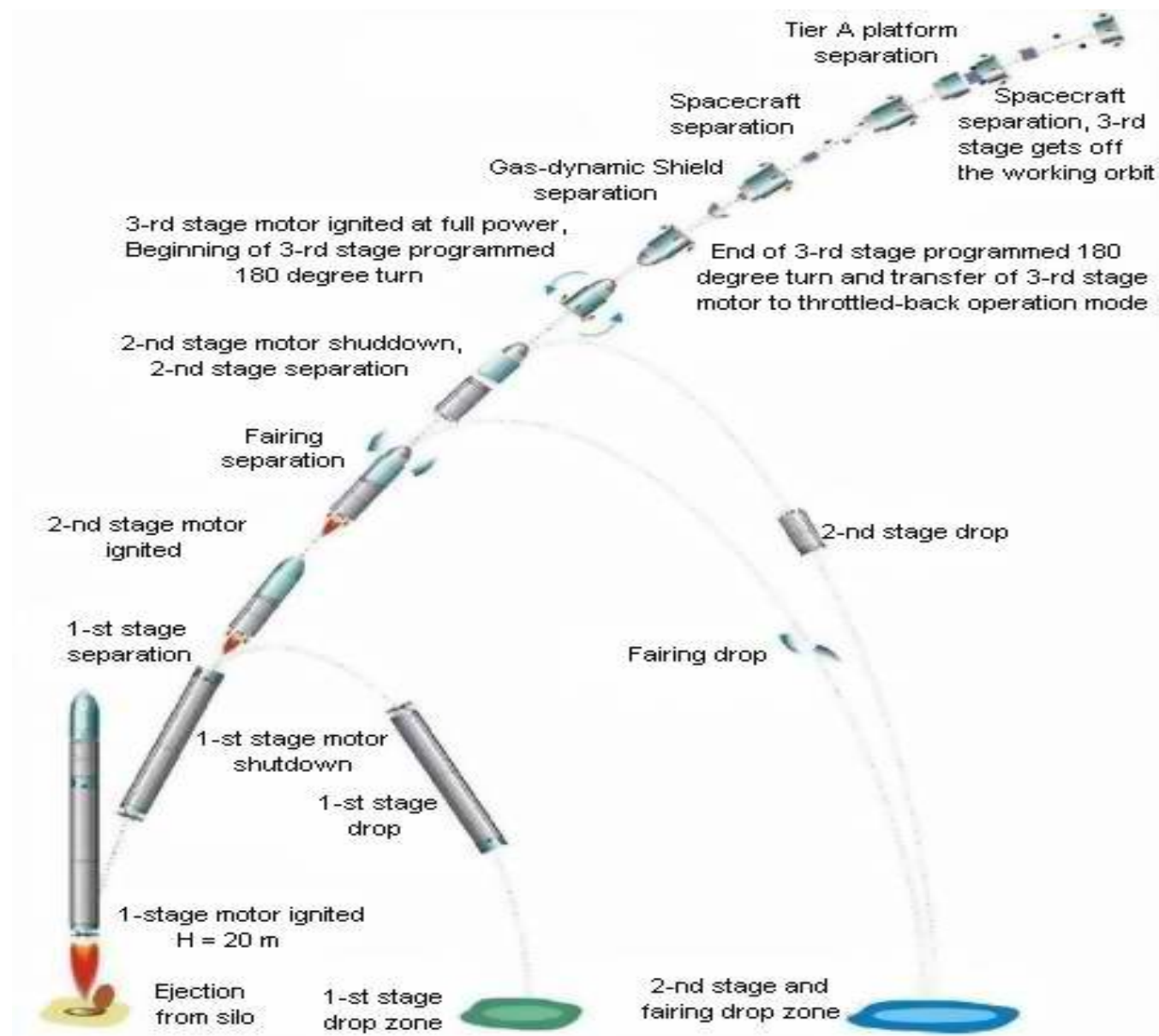
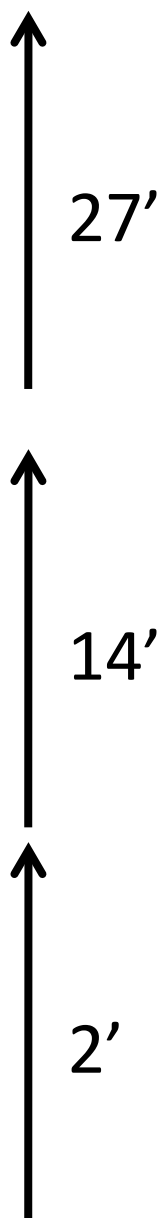
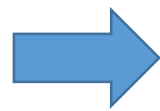
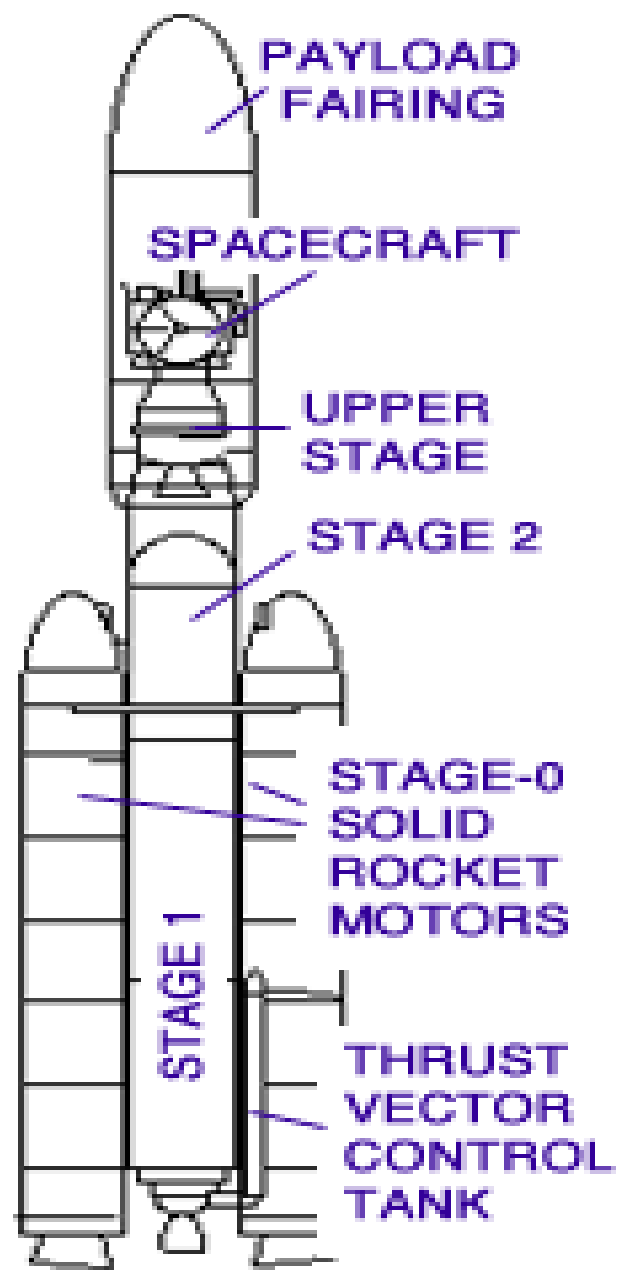
ERS-SAR has a Revisit Time
~ 16 days

A launcher puts a satellite into orbit.
Remote sensing satellites are usually launched by an Expendable Launch Vehicle (ELV) that can be used only once.
The Space Shuttle is a reusable launch system, i.e., vehicle is launched and recovered more than once.



Ariane 5 at the Kourou launch base
(French Guyana)



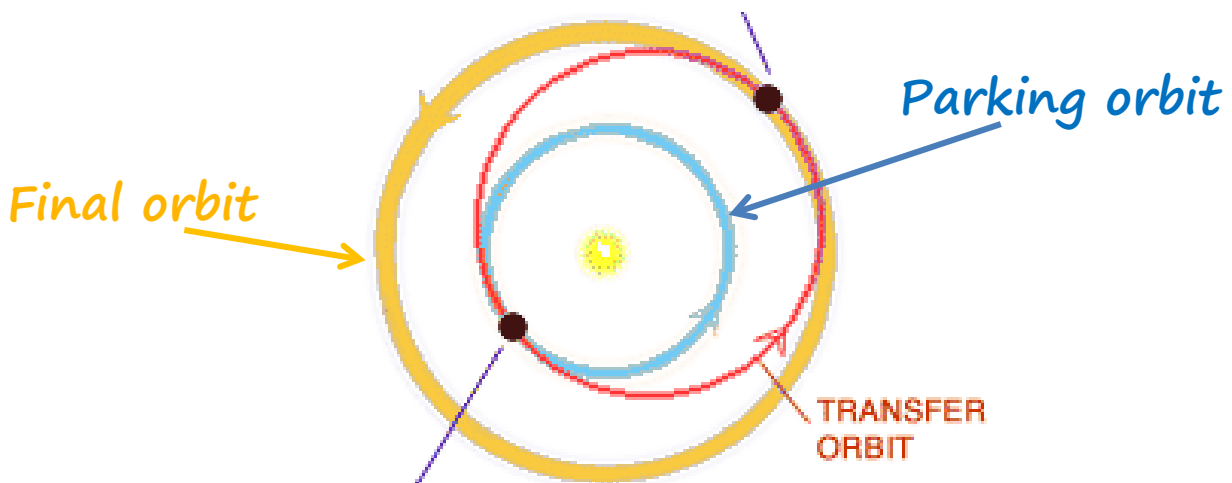


Orbit ascent can be performed through a direct or indirect launch.

In the latter case, satellites do not reach their orbit directly but they are put into a **parking orbit** and then, with on-board engines, they are transferred into their **final orbit**. This transfer takes place in two phases and it is called

Hohmann Transfer

Apogee of the Transfer Orbit



Perigee of the Transfer Orbit

The parking orbit is a circular orbit with radius r_1

Here the satellite has velocity $v_1 = \sqrt{\frac{\mu}{r_1}}$

The final orbit is a circular orbit with radius r_2

and velocity $v_2 = \sqrt{\frac{\mu}{r_2}}$

The transfer orbit is an elliptical orbit with Perigee = r_1 and Apogee = r_2

In the elliptical orbit, velocity at perigee is $v_p = \sqrt{\frac{2\mu r_2}{r_1(r_2 + r_1)}}$

Giving a velocity variation $\Delta v_1 = v_p - v_1$, the orbit becomes elliptical with $v_a = \sqrt{\frac{2\mu r_1}{r_2(r_2 + r_1)}}$

With $\Delta v_2 = v_2 - v_a$ the orbit becomes circular

The total impressed velocity variation is

$$\Delta v_{tot} = \Delta v_1 + \Delta v_2$$

that is proportional to the transfer cost, i.e., to the fuel consumption.

It is possible to demonstrate that the Hohmann transfer minimizes the Δv_{tot} necessary to the manoeuvre, as long as the final orbit has a radius lower than 11 times the Earth radius.

The weight of the satellite to be launched is very important:

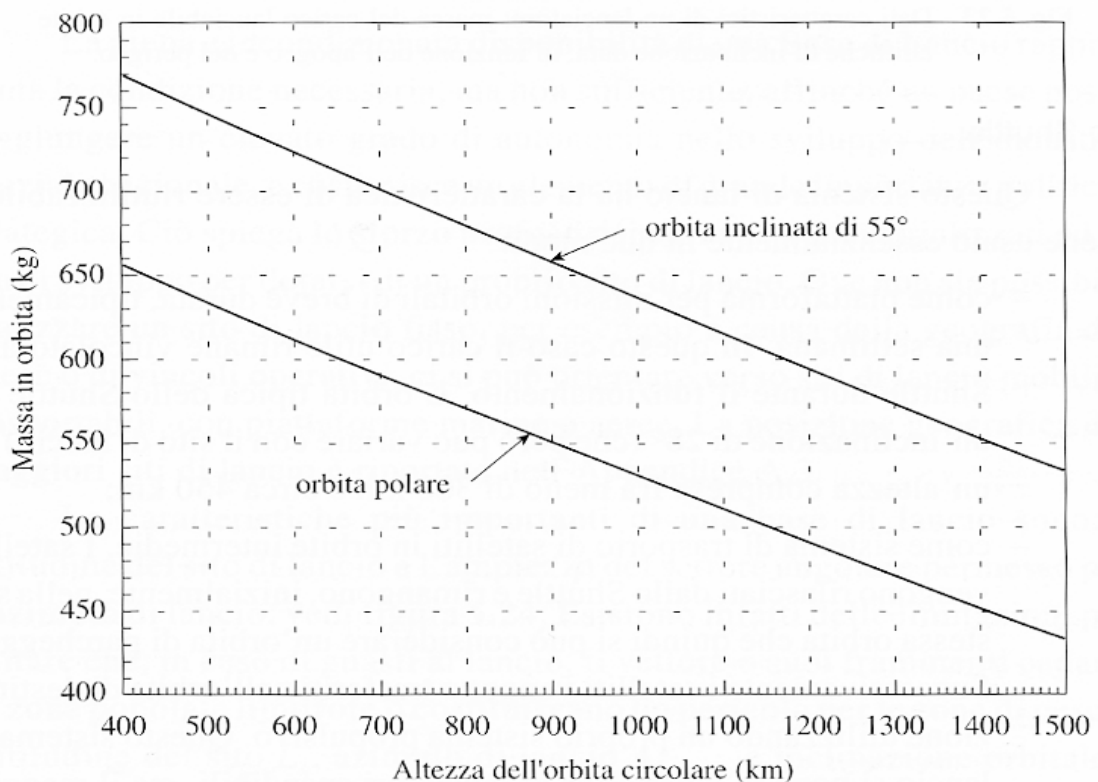
The heavier the satellite,
more fuel is necessary to put it into orbit,
but this means larger tanks in the launcher,
and consequently more weight to be launched.

The weight of the satellite includes the weight of the fuel necessary to the transfer manoeuvre.

Weight of the payload =

Satellite Weight – Fuel Weight

The higher the orbit, the larger quantity of fuel is necessary to reach its altitude, and therefore the lower is the weight of the payload.



Launchers are compared on the basis of the weight of the payload that they can launch into a reference orbit (GTO)

The Geostationary Transfer Orbit has perigee at ~ 600 Km and apogee at ~ 36000 Km

Delta (2tons)

Titan (4.5 tons)

Ariane 5(6.5 tons)

STS(30 tons in LEO)

The geographical features of the launch base set limits to the launch itself.

Orbit inclination and latitude λ of the base are linked by

$$\cos i = \sin \varphi \cdot \cos \lambda$$

φ is the angle between the velocity vector of the satellite and the Earth meridian at the launch base.

If there were no restrictions on the launch azimuth φ , from a given base it would be possible to reach every inclination between 90° and λ .

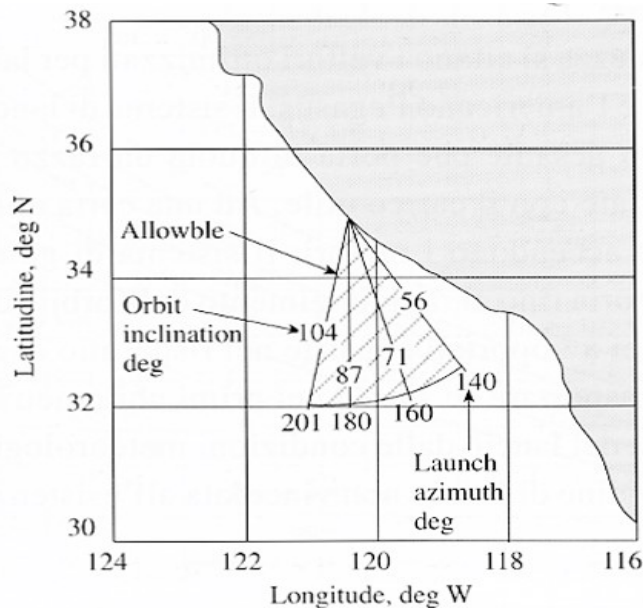
If $\varphi=90^\circ$, the orbit inclination coincides with the base latitude.

But φ is constraint by the proximity of built-up areas.

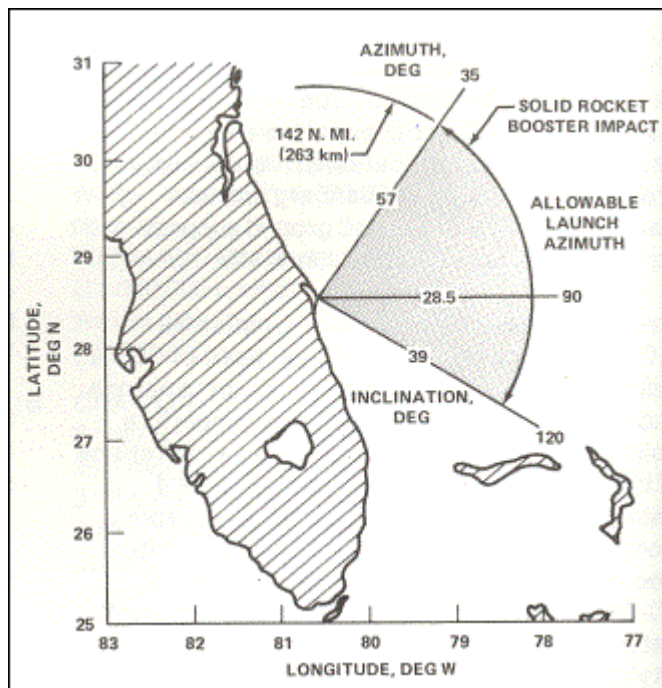
Usually, the launch is eastward so that the Earth rotation thrust is exploited.

Launch Azimuth constraints

Vandenberg Air Force Base (California)



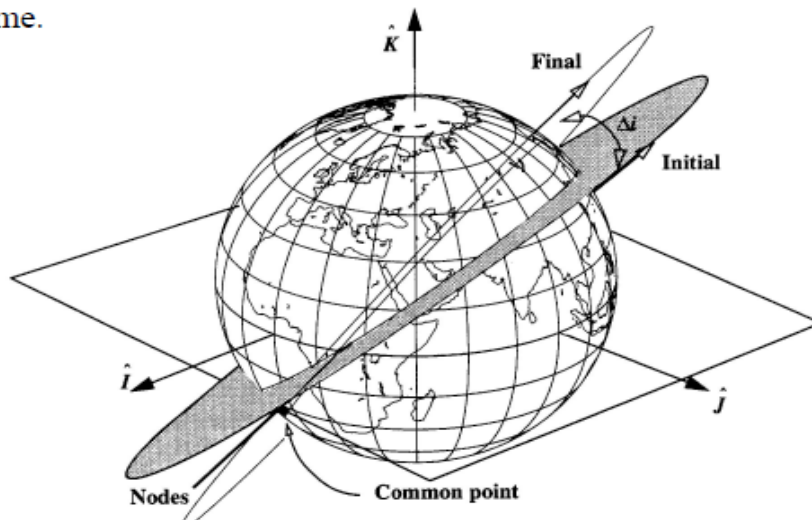
Kennedy Space Center (Cape Canaveral, FL)



Inclination-Only Changes

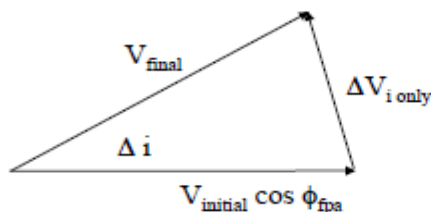
Must occur at one of the nodes (only common points).

Magnitudes of velocity vectors (initial, final) must remain the same.



Inclination-Only Changes

Check both nodes to see which is less Δv , although for circular orbits there is no difference. Only need ϕ_{fpa} for elliptical orbits. Remember, plane changes are expensive.



Isosceles
triangle

$$\sin\left(\frac{\Delta i}{2}\right) = \frac{\Delta v_{i \text{ only}}}{2v_{\text{initial}} \cos(\phi_{fpa})} \Rightarrow \Delta v_{i \text{ only}} = 2v_{\text{initial}} \cos(\phi_{fpa}) \sin\left(\frac{\Delta i}{2}\right)$$

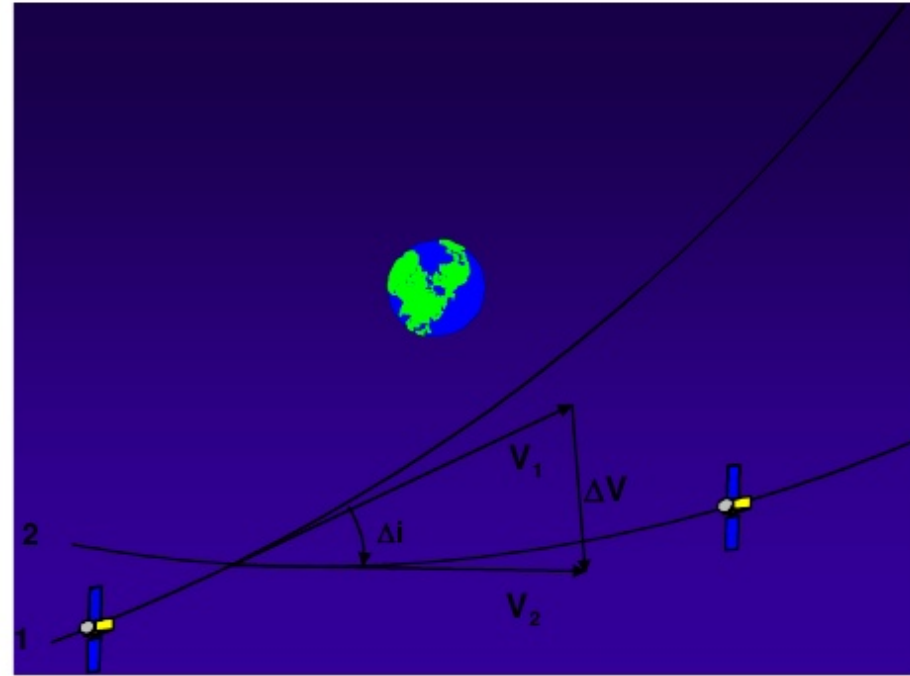


4. Orbit Change Maneuvers

Out-Of-Plane Maneuver

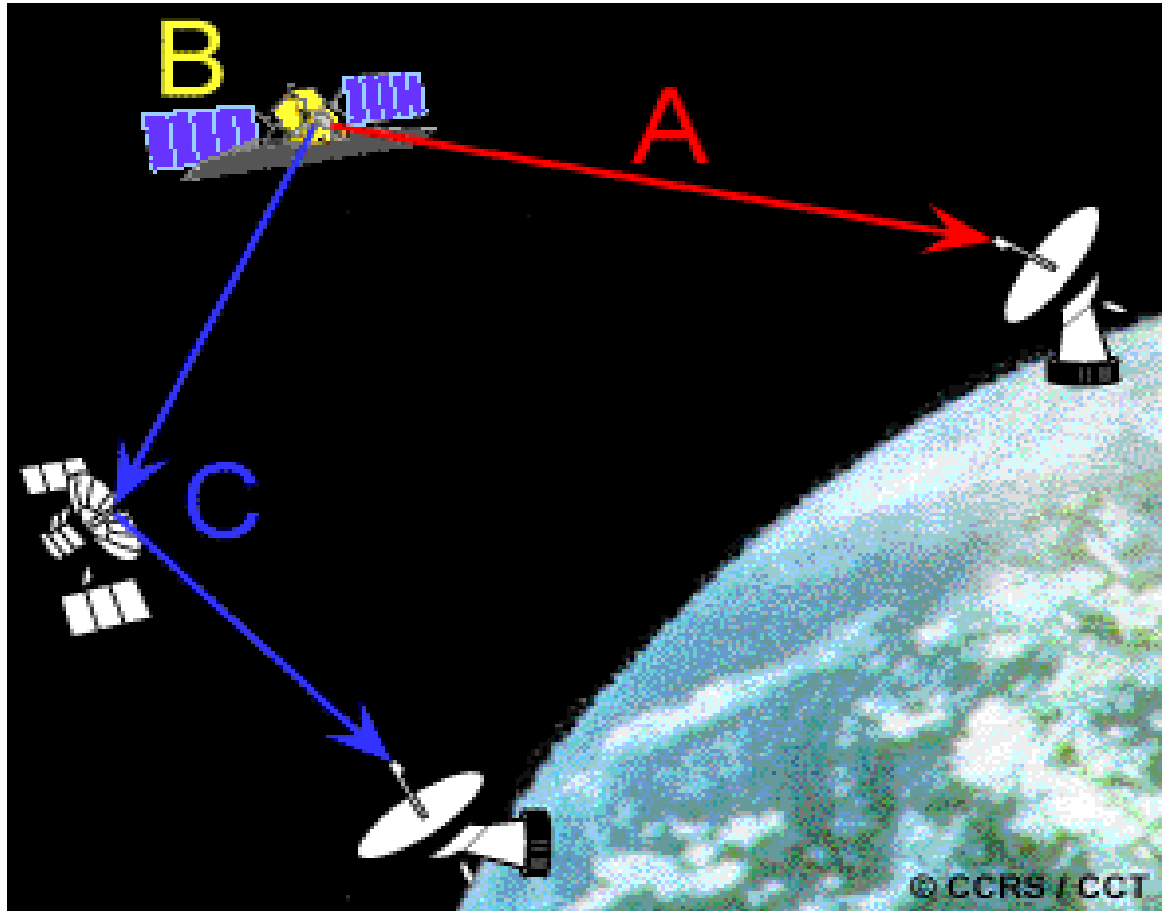
$$|\vec{v}_1| = |\vec{v}_2| = v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

$$\Delta v = 2 \cdot v \cdot \sin \frac{\Delta i}{2}$$



(In order to change Earth inclination when the satellite is in orbit, the direction of velocity is changed, leaving its module unaltered)



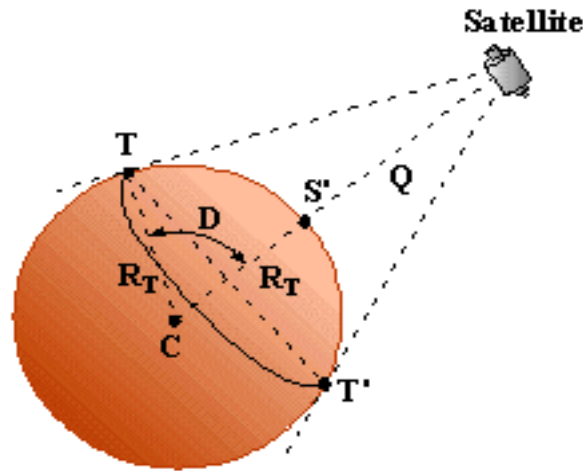


Data acquired by the instruments on a satellite can be

- transmitted directly to the Earth (A),
- temporarily stored on board (B) and then transmitted to Earth,
- transmitted to another satellite (C)

(Tracking and Data Relay Satellite System = satellite system for Communication in geostationary orbit)

The acquisition area is the segment of a sphere on the Earth surface that is obtained tracing the tangents from the satellite to the Earth.



Subsatellite point S' and area of acquisition

The radius of the acquisition area is

$$S'T = R_T \cdot D = R_T \cdot \cos^{-1} \frac{R_T}{R_T + Q}$$

In order to receive the signals from a GEO satellite, the Earth station must be inside the acquisition area.

In the case of a LEO satellite, the acquisition area is built fixing the subsatellite point S' at the Earth station.

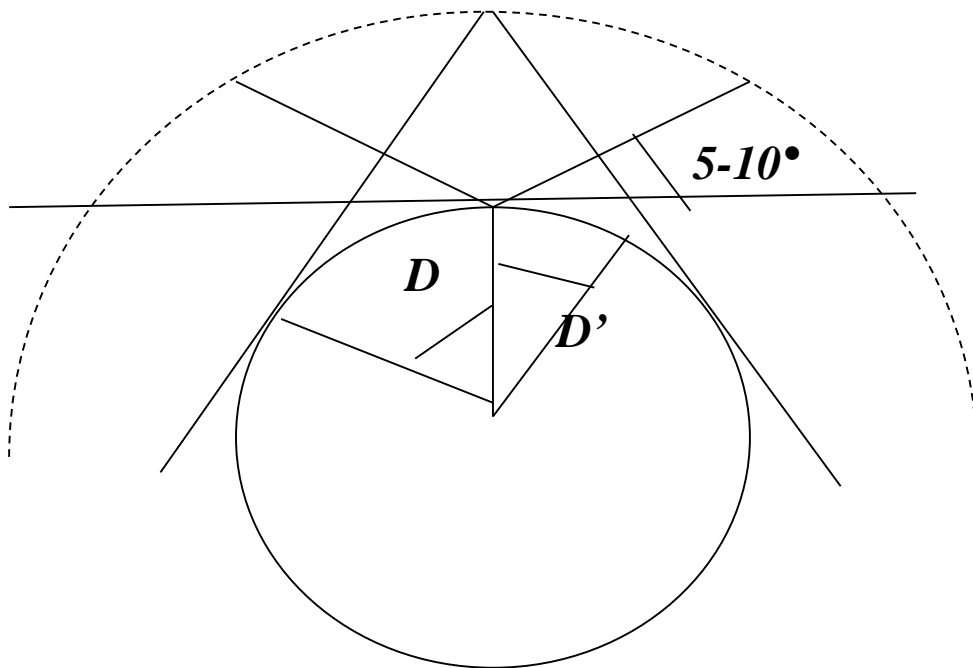
The satellite maintains in touch with the Earth station as long as the nadir track lies inside the acquisition area.

LEO Satellites ($D \sim 28^\circ$) $S'T \approx 3000 \text{ Km}$

GEO Satellites ($D \sim 81^\circ$) $S'T = 9000 \text{ Km}$

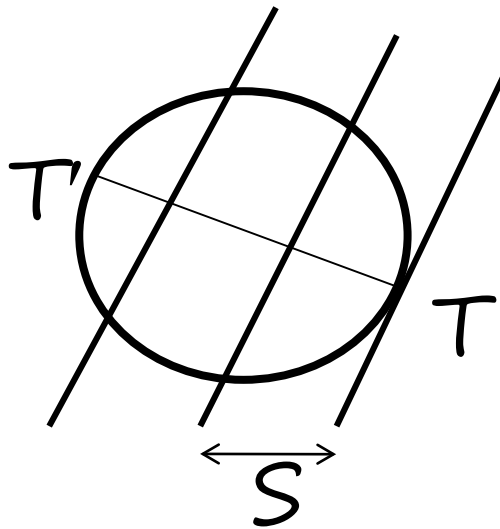
Actually, the acquisition area corresponds to an angle $D' < D$ that corresponds to the minimum height that a satellite must have on the horizon in order to be observed radio-electrically.

Usually, a minimum angle on the horizon between 5° and 10° is considered.



LEO satellites crosses many times the acquisition area.

The number of satellite passes visible in the acquisition area is equal to the number of fundamental intervals contained in the diameter of the acquisition area.

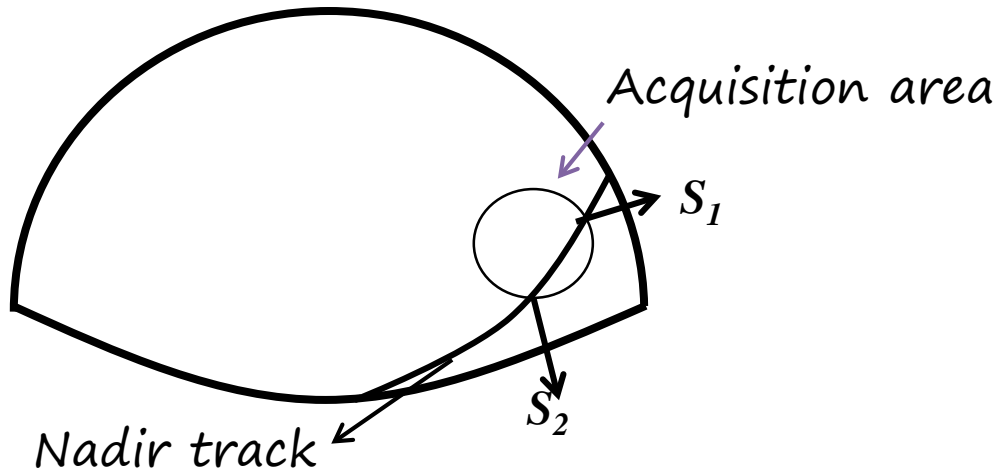


$$n = \frac{TT}{S \cos \lambda}$$

S is the fundamental interval in Km
at the equator

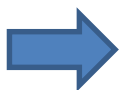
λ is the latitude of the Earth station

The time length of the satellite overpass depends on the length of the nadir track that falls inside the acquisition area, and on the satellite velocity



$$\Delta t = S_1 S_2 \cdot \frac{T}{360}$$

$S_1 S_2$ is given in degrees, and can be
at maximum $= 2D'$



16 settembre ore 7.56



16 settembre ore 9.35



Stazioni a terra dell'ESA

Kiruna (Svezia)

Kourou(Guyana Francese)

Malindi (Kenia)

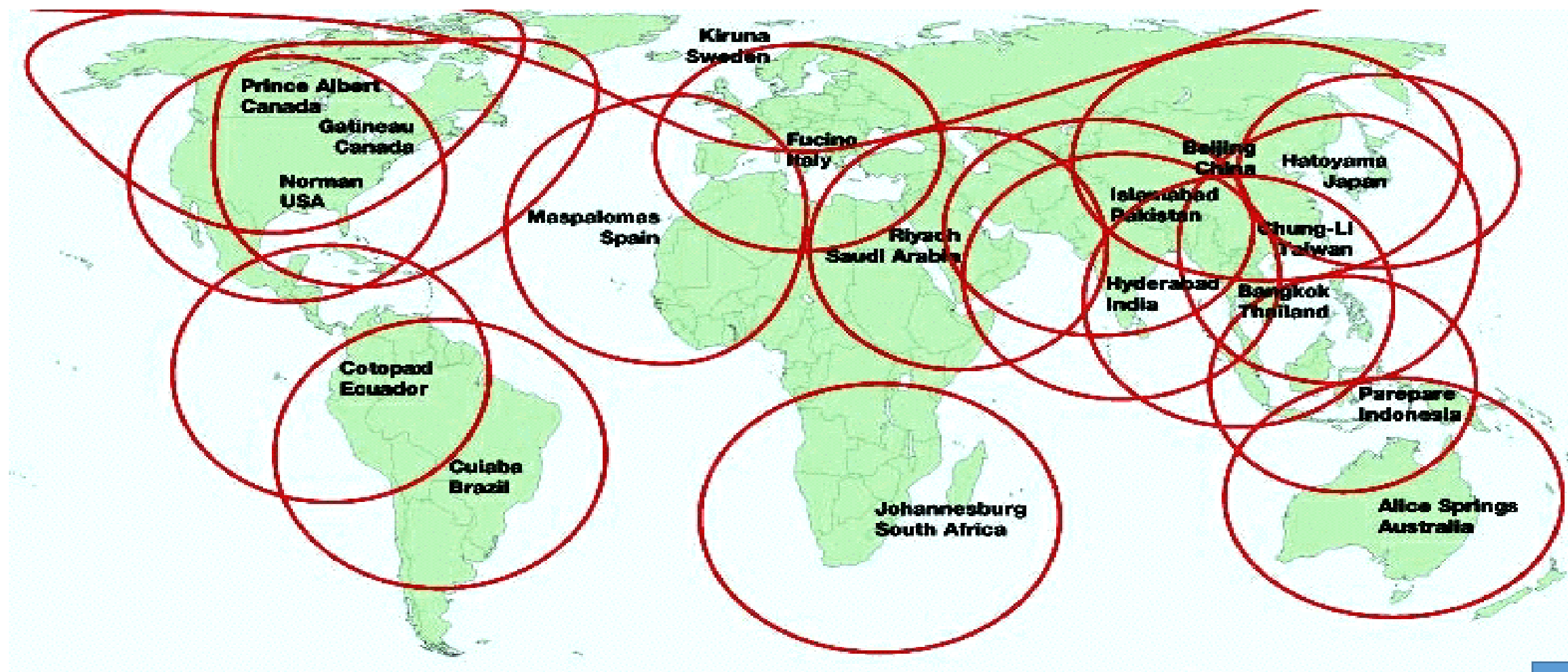
Maspalomas (Gran Canaria)

Perth (Australia)

Redu (Belgio)

Villafranca (Spagna)

Le stazioni Landsat nel mondo



Molniya Orbits

Very eccentric and inclined, with Apogee in the Northern hemisphere. At Apogee the satellite slows down, so that it appears almost stationary for a long time

$$h_A = 39420 \text{ km}$$

$$h_P = 1000 \text{ km}$$

$$T \sim 12 \text{ h}$$

$$e = 0.72$$

$$i = 63.4^\circ$$

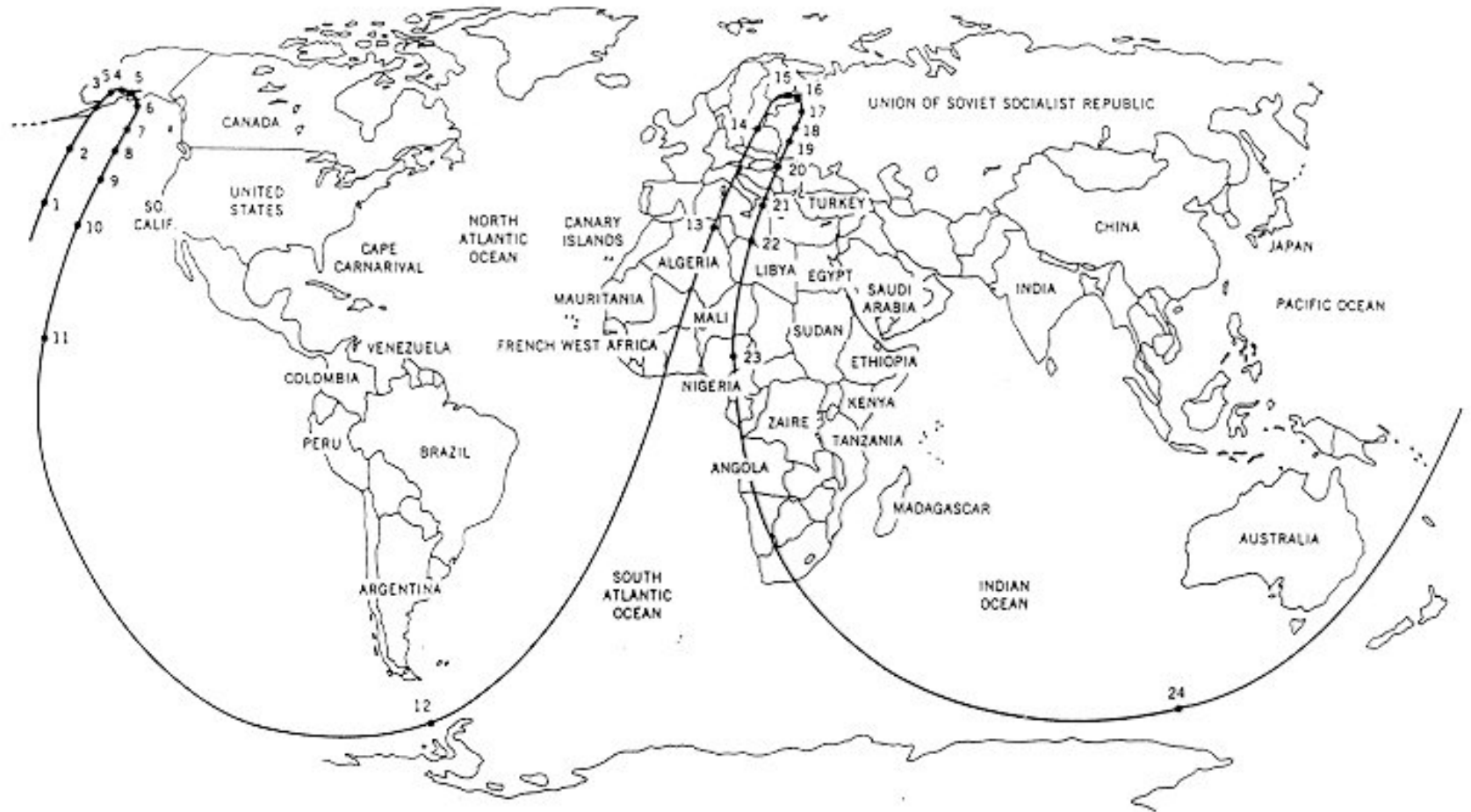
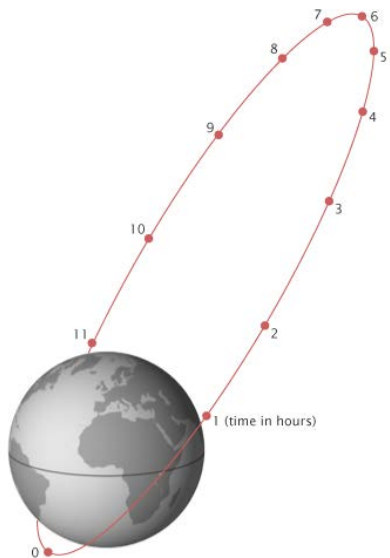


Figure B-10. Ground trace for a 12-hour elliptical orbit with an ellipticity of 0.7 and inclination of 63.4° .

Details on Space debris

- [http://www.esa.int/spaceinvideos/Videos/2013/04/Space debris story](http://www.esa.int/spaceinvideos/Videos/2013/04/Space_debris_story)
- <http://orbitaldebris.jsc.nasa.gov/index.html>
- Documented collisions:
- February 2009: LEO, American commercial satellite collided with defunct Russian military satellite Kosmos-2251 -> created 2000 pieces of debris
- In 2009 five satellite manouvers had to be done to avoid further collisions with the fragments: ACQUA, LANDSAT 7 (700km), Space Station, Space Shuttle 400 km, NASA tracking and Data Relay Satellite.
- Recent measures: satellites should be brought down to sufficient low orbit (example several 100km) in order to decelerate in a decade of years and reenter the atmosphere